

A Supermodular Location Game

Two players simultaneously choose possibly random dollar bets $x, y \in [0, 4]$, and each earns payoffs $u(x, y) = 12x - 11x^2 + 12xy^2 - 2xy^3$.

1. Do there exist any strictly mixed Nash equilibria?
2. Is this a supermodular game?
3. Find all symmetric Nash equilibria.

Hint: $6 - 11z + 6z^2 - z^3 \equiv (3 - z)(2 - z)(1 - z)$

4. Find all stable symmetric Nash equilibria. (Be careful!)
5. What strategies are strictly dominated? Intuit which strategies are rationalizable. Argue formally, if you can, using monotonicity of a partial derivative of $u(x, y)$.
6. Bonus: Can you add in a payoff parameter that produces amplified comparative statics?