Econ 809 Problem Set 1 (due Wed after break)

PS This is the basic tools HW. Future HW's have applied research problems to ponder.

1. Assume a partial order on integers, where a<=b if a is a factor of b. What do we call the meet and join of two numbers?

2. Assume a partial order of set inclusion on the set of all subsets of a finite set. Assume each element is a college you might apply to, k=1,2,…,N. Assume the function f(S) is the probability of gaining admission to some college s into the set S \subset N.
(a) What is the smallest sublattice that contains any three given schools I,j,k?
(b) Assume admission to college k has chance p_k, and admission events are independent. Is f supermodular, submodular or neither? Prove your claim as succinctly as you can.

3. Prove that the 0-1 indicator function I of
(a) intervals (-infty,a] or [a,infty) are log-supermodular on the reals
(b) rectangles [0,a] ×[0,b] are log-supermodular in the northeast partial order in $[0,1]^2$

4. Prove that $|X-Y|^{\gamma}$ is submodular if $\gamma \geq 1$.

5. Suppose every player i=1,2,….,n+1 picks a random time t according to a cdf F(t) over [0,infty). Assume n players independently make their time choice. If your time is the kth highest you get payoff x_k, where $x_k - x_{k-1}$ changes sign m<n times. Show that every player's expected payoff changes sign at most m times in t.