Bluffing Question on 2021 Prelim SOLVED

The Economics of Bluffing (Typos fixed). In a card game, Rowena is dealt a high or low card, beating Colin iff she has a high card. But someone may fold before this winner is known. There is a prior forced initial ante $A > 0$, not modeled as a choice (and so its payoff does not matter, if it is lost, but will matter if it is won).

Two risk neutral players Rowena and Colin have utils, or "payoffs", measured in dollars. Simultaneously, Rowena and Colin each folds or bets an extra fixed bet $B > A$. Rowena's card is either high or low. If both players fold, then each gets payoff 0. A player who bets when his opponent folds wins both antes (payoff $2A$), and his opponent's payoff is 0. If both players bet, then payoffs are (i) $B + 2A$ for Rowena and $-B$ for Colin if Rowena's card is high, and (ii) $B + 2A$ for Colin and $-B$ for Rowena if her card is low.

1. First assume that Rowena's card is commonly seen, chosen to be high by Nature with chance $p \in [0, 1]$. Apply dominance logic to find all Nash equilibria of the game. [2]

   Solution of the Motivated Problem, with a loss of B when you bet and lose: If the card were known, dominance logic gives:

   - Rowena low card → Colin always bets → Rowena always folds
   - Rowena high card → Rowena always bets → Colin always folds

	Fold	Bet
Fold	0,0	0,2A
Bet	2A,0	-B,B+2A

   Bad card (1-p)

	Fold	Bet
Fold	0,0	0,2A
Bet	2A,0	B+2A,-B

   Good card (p)

2. Now assume that neither Rowena nor Colin see Rowena's card. Assume $p \in [0, 1]$ is commonly known. Set this up as a Bayesian normal form game. [1]

   Solution: Risk neutrality, and the single information set for each player, implies that both players project the strategic situation into the following normal form game:

	Fold	Bet
Fold	0,0	0,2A
Bet	2A,0	(2p-1)B+2pA,(1-2p)B+2(1-p)A

3. Find all Bayesian Nash equilibria. Plot Rowena's equilibrium payoff as a function of p. [3]

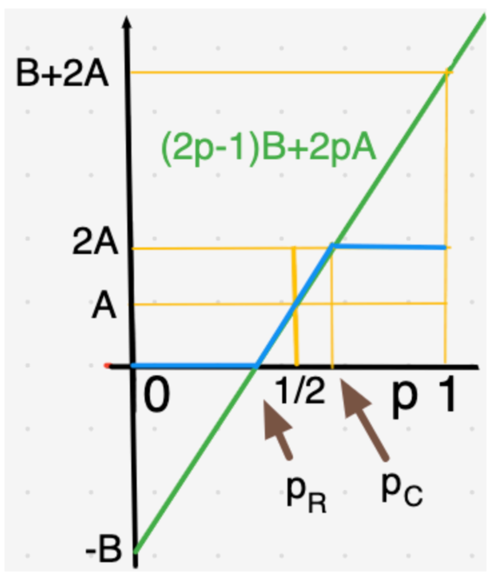
Solution: In the Bayesian game, we have

- Betting is strictly dominant for Colin iff p is not too big: $(1-2p)B+2(1-p)A > 0$
  - Inequality binds at p_C, namely, $(1-2p_C)B + 2(1-p_C)A = 0$
- Betting is strictly dominant for Rowena iff p is not too small $(2p-1)B+2pA > 0$
  - Inequality binds at p_R, namely, $(2p_R - 1)B + 2p_R A = 0$

As $1 > p_C = (B + 2A)/(2B + 2A) > 1/2 > B/[2(A + B)] = p_R > 0$, betting is always dominant for at least one player:

- $p < p_C$: Betting is strictly dominant for Colin
- $p > p_R$: Betting is strictly dominant for Rowena

Altogether, Rowena's equilibrium payoff is unique and its expected payoff is the blue line:



4. Finally, assume that Rowena sees her card, and Colin does not, and the chance p that it is high is common knowledge. Before playing the game, let Rowena send an imperfect cheap talk signal. Characterize the best verifiable bluffing signal, meaning that she pre-commits to a rule: when her card is high, she says it is high, but when it is low, she still says it is high with a pre-committed chance q, and this is the best such signal. [4]

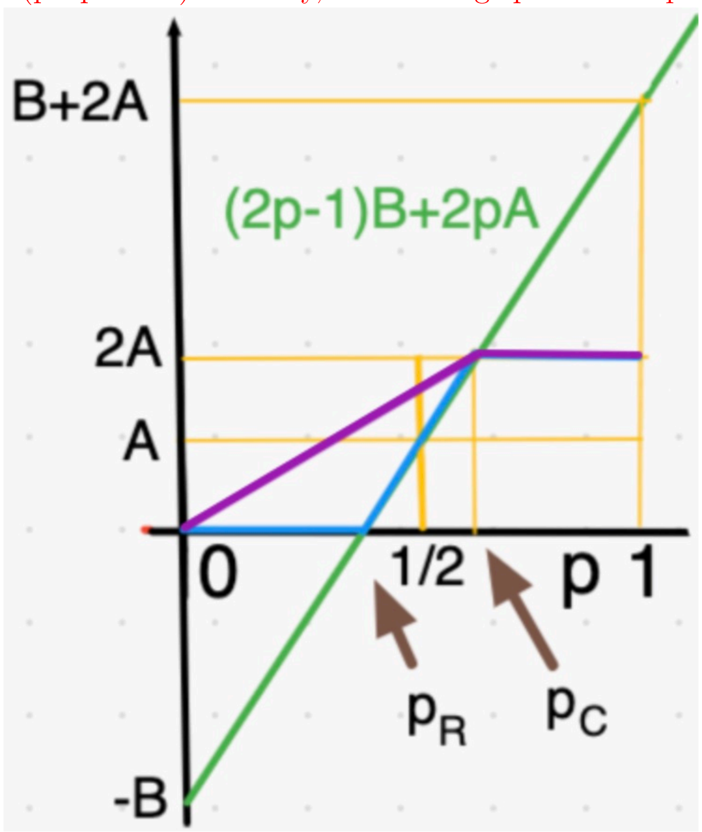
   (a) Plot Rowena's payoff from optimally bluffing as a function of $p \in [0, 1]$, adding the earlier plot.

   (b) Compute the chance q of bluffing with a low card. How does q change with the prior probability p of a high card?

   (c) Intuitively explain why bluffing is profitable for Rowena.

Solution: This bluffing with commitment rule is an example of Bayesian persuasion. It is useful if you are repeatedly playing cards with the same players, and so your bets always create the same beliefs, or the eventually the other players would learn that your bluff was not credible. For indeed, gamblers have reputations to consider.

(a) Rowena always bets if $p \geq p_C$, since folding is a dominant strategy. But she uses a bluffing strategy $p < p_C$ to attain the least concave function over the blue equilibrium payoff function (purple line). Namely, her bluffing spreads the posterior to 0 or p_C.



(b) Think of the cheap talk as a Rowena truthfully announcing whether she will bet or fold in the simultaneous move game. With a high card, Rowena always announces a plan to bet, but with a low card, announces a plan to bet with chance $q = [p/(1-p)](1-p_C)/p_C$. This bluffing strategy ensures that the posterior odds after seeing a bet are:

$$p_C/(1-p_C) = P(good|bet)/P(bad|bet) = [p/(1-p)][1/q]$$

As the prior probability p of a high card increases on $[0, p_C]$, the chance q of bluffing rises from 0 to 1.

(c) The idea behind bluffing strategy is that even though $p < p_C$, Rowena can sometimes induce Colin to fold when she bets, since it raises the posterior from p to p_C.