Assignment #2

1. (easy) Assume two states L&H, with posterior r=0.5 leaving indifference between symmetric two actions. Assume the signal realization has density f^H(s)=6(s-s^2) and f^L(s)=1 on [0,1].

(a) What is the value of one signal, given a prior p on H? Assume payoff max(2r,1-2r) given posterior r on H.

Solution: f^H(s) =6(s-s^2) has a max when 1=2s, or s=1/2, when f^H(1/2)=3/2. Value of signal is E max(2R,1-2R). To be clear, the respective payoffs of actions A & B are (0,2) & (1,-1) in states L & H, with indifference at r=1/4. Given prior p, the threshold signal yielding indifference is

$$(1-p)/p= f^H(s)/f^L(s)= 6(s-s^2) \qquad (**)$$

Solving this as a quadratic equation yields s^2–s + (1–p)/(6p) =0. The two roots are s(p)= {1±√[1–4(1–p)/(6p)]}/2 = 1/2± (1/2) √[(5p–2)/(3p)]

Assume p≤2/5. Always take action A. So information has zero value: it never impacts decisions.

Assume 2/5<p<1. There are two roots of (**) – i.e., $0<s_1(p)<1/2<s_2(p)<1$, symmetric about ½, or equivalently, $s_1(p)+s_2(p)=1$. Signals in $(s_1(p),s_2(p))$ favor taking the high action, say B, and action A for all other signal outcomes. So the **value of information** (VoI) is the *difference*

p{Expected payoff in state H} + (1-p){Expected payoff in state L} – max (2p,1–2p)
$= p[F^H(s_2(p))–F^H(s_1(p))]+ (1–p)\{2[F^L(s_2(p))–F^L(s_1(p))]–[1–F^L(s_2(p))+F^L(s_1(p))]\}$ – max (2p,1–2p)
$= p[F^H(s_2(p)) –F^H(s_1(p))]+ (1–p)[3F^L(s_2(p)) –2F^L(s_1(p)) –1]$ – max (2p,1–2p)

Now, $F^H(s)$ = 3s^2 – 2s^3 = s^2+2s(s-s^2) =s–(1–p)/(6p)+ s(1-p)/(3p) = s(1+2p)/(3p) - (1-p)/(6p)
➔ $F^H(s_2(p)) – F^H(s_1(p)) = [s_2(p) –s_1(p)]$ (1+2p)/(3p) = √[(5p–2)/(3p)] (1+2p)/(3p)
and $3 F^L(s_2(p)) –2F^L(s_1(p))-1 = 3s_2(p) –2s_1(p) -1$= (5/2) √[(5p–2)/(3p)] –½
➔ VoI = p√[(5p–2)/(3p)] (1+2p)/(3p)+ (1–p)[(5/2) [√[(5p–2)/(3p)] –½] – max (2p,1–2p)

(b) In what state L or H or both or neither will informational herding lead to everyone eventually choosing the correct action?

Solution: Since the private signal likelihood ratio f^H(s)/f^L(s)= 6(s-s^2) is boundedly finite but tends to 0 near 0 & 1, we learn the truth in state L, but do not in state H with positive probability.

(c) Find all cascade sets that are nonempty intervals of public beliefs.

Solution: As noted, if p≤2/5, then we ignore all private signals (so a cascade).