# *Constrained Efficient Informational Herding: An Introduction to Optimal Experimentation*

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April 2, 2025

# Social Learning — Case 2: The Altruistic Professor

- Altruistic Professor's payoff is the expected Student's payoff.
- When $\pi \leq 1/3$, the selfish Professor always chooses paper $\ell$
- To help Student, altruistic Professor leans against prior belief, by choosing a lower *altruistic threshold signal* $\hat{\sigma} < \bar{\sigma}(\pi)$.
- Paper $\ell$ integrates over private signals $\sigma < \hat{\sigma}$, and $h$ over $\sigma > \hat{\sigma}$
- Seeing Professor's actions, Student updates to posterior beliefs

$$\hat{p}_\ell(\pi, \hat{\sigma}) = \frac{\pi\hat{\sigma}^2}{\pi\hat{\sigma}^2 + (1-\pi)\hat{\sigma}} < \pi < \hat{p}_h(\pi, \hat{\sigma}) = \frac{\pi[1-\hat{\sigma}^2]}{\pi[1-\hat{\sigma}^2] + (1-\pi)[1-\hat{\sigma}]}$$

  - Proof: Use Bayes rule, given $f^H(\sigma) = 2\sigma$ & $f^L(\sigma) = 1$.
  - See Smith, Sorensen, Tian, (2020) = SST
- Since $E[P|\pi, \hat{\sigma}] = \pi$, by the law of iterated expectations:

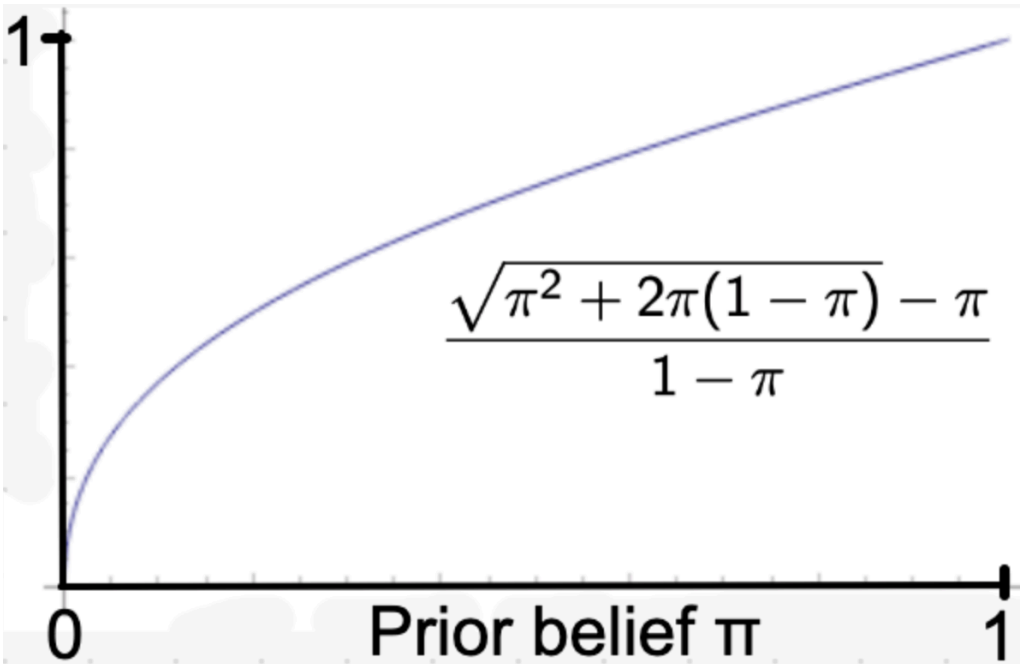
$$E[V(P)|\pi, \hat{\sigma}] = E[(5P - 4 + 1/P)/2|\pi, \hat{\sigma}] = \tfrac{5}{2}\pi - 2 + \tfrac{1}{2}E[(1/P)|\pi, \hat{\sigma}]$$

- Maximize this over $\hat{\sigma}$
  - Algebra is in Smith, Sorensen and Tian (2021), "Informational Herding, Optimal Experimentation, and Contrarianism"

# The Altruistic Professor's Contrarian Behavior

- Selfish Professor's *posterior odds threshold* was 1
- As confidence in state $H$ rises ($\pi \uparrow$), Altruistic Professor leans more against paper $h$, i.e. posterior odds threshold increases
- Our altruistic threshold signal $\Rightarrow$ posterior odds threshold:

$$\frac{f^H(\hat{\sigma}(\pi))}{f^L(\hat{\sigma}(\pi))}\frac{\pi}{1-\pi} = 2\hat{\sigma}(\pi)\frac{\pi}{1-\pi} = \frac{\sqrt{\pi^2+2\pi(1-\pi)}-\pi}{1-\pi}.$$

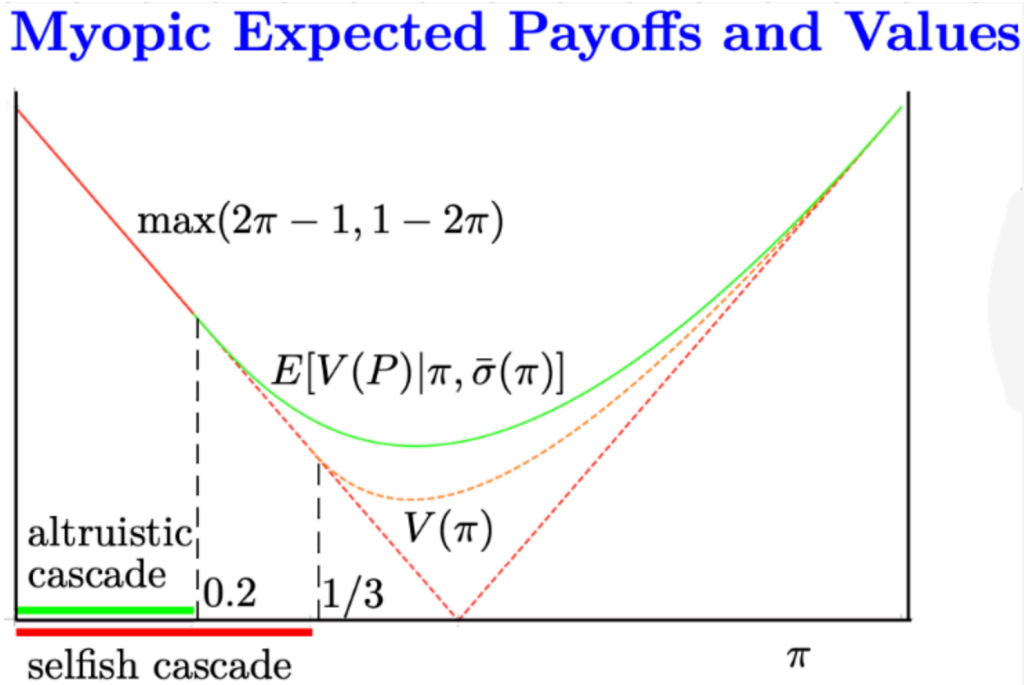



# The Altruistic Professor's Optimal Value

- The altruistic Professor's value function is the myopic payoff $U(\pi)$ on $[0, 1/5]$ inside the cascade, and on $[1/5, 1]$ is:

$$E[V(P)|\pi, \hat{\sigma}(\pi)] = V(\pi) + \frac{1-\pi}{\pi} \cdot \frac{\pi(3-2\pi) - \sqrt{1-(1-\pi)^2}}{\pi + \sqrt{1-(1-\pi)^2}}.$$

- *Altruism shrinks Professor's cascade beliefs:* $[0, \frac{1}{3}] \to [0, \frac{1}{5}]$.




- Least signal LR $f^H(0)/f^L(0) = 0 \Rightarrow$ No cascade on paper $h$
- Max signal LR $f^H(1)/f^L(1) = 2 < \infty \Rightarrow \exists$ cascade on paper $\ell$