An Economic Theory Masterclass

Part V: Public Goods

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Public Goods Taxonomy

- *Rival* good: one consumer's use reduces another's benefit
- *Nonrival* good: no consumer's use reduces another's benefit
- *Excludable* / *nonexcludable* good: one can / cannot prevent others from jointly consuming a unit of the good

Goods	**Rival**	**Nonrival**
Excludable	Private good	Club good
Nonexcludable	Congestion public good	Pure public good

Examples of Congestion Public Goods

- City roads, wifi, internet traffic, water out West

GLOBAL APPLICATION TOTAL TRAFFIC SHARE

1. **YOUTUBE:** 2019: 8.69% **2020: 15.94% (+7.25%)**
2. **NETFLIX:** 2019: 12.87% **2020: 11.42% (-1.45%)**
3. **HTTP:** 2019: 3.61% **2020: 6.57% (-2.96%)**
4. **BITTORRENT:** 2019: 7.75%% **2020: 5.23% (-2.52%)**
5. **FACEBOOK:** 2019: 3.37% **2020: 3.68% (+0.37%)**
6. **HTTP MEDIA STREAM:** 2019: 13.76% **2020: 3.64% (-10.12%)**
7. **GOOGLE:** 2019: 1.23% **2020: 2.91% (+1.68%)**
8. **WORDPRESS:** 2019: 0.10% **2020: 2.88% (+2.78%)**
9. **INSTAGRAM:** 2019: 2.64% **2020: 2.72% (+0.08%)**
10. **FACEBOOK VIDEO:** 2019: 2.46% **2020: 2.29% (+0.17%)**

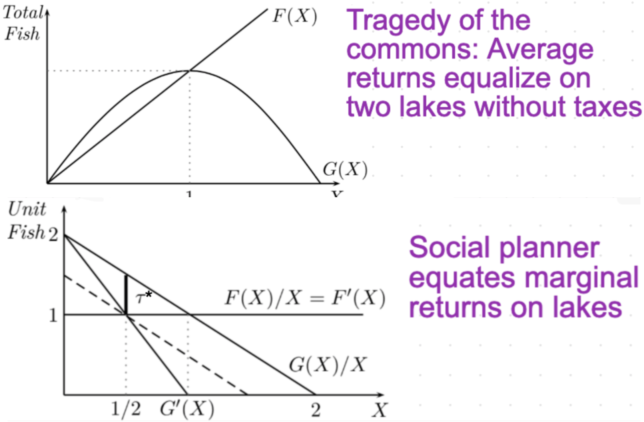
GLOBAL APPLICATION CATEGORY TOTAL TRAFFIC SHARE

1. **VIDEO STREAMING:** 2019: 55.44% **2020: 57.64% (+2.20%)**
2. **SOCIAL NETWORKING:** 2019: 8.95% **2020: 10.73% (+1.78%)**
3. **WEB:** 2019: 10.14% **2020: 8.05% (-2.09%)**
4. **MARKETPLACE:** 2019: 5.90% **2020: 4.97% (-0.93%)**
5. **MESSAGING:** 2019: 3.79% **2020: 4.94% (+1.15%)**
6. **FILE SHARING:** 2019: 8.51% **2020: 4.64% (-3.87%)**
7. **GAMING:** 2019: 2.20% **2020: 4.24% (+2.04%%)**
8. **VPN: 2.56%** 2019: 2.46% **2020: (+0.10%)**
9. **CLOUD:** 2019: 1.26% **2020: 1.83% (+0.57%)**
10. **AUDIO:** 2019: 55.44% **2020: 0.39% (-0.39%)**

The Tragedy of the Commons

- Public areas (e.g. air) lack property rights $\Rightarrow$ disasters
- The commons lacks well-defined property rights (Coase fails)
- Continuum mass of fishermen each allocates hours X_A, X_B between Lakes A and B, where $X_A + X_B = \bar{X} > 1$.
- Lake A has constant returns: $F(X_A) = X_A$
- Lake B has decreasing returns: $G(X_B) = 2X_B - X_B^2$
- Every fisherman faces a binary choice: Lake A or Lake B
  - A fisherman chooses the lake with the higher expected return.
  - There is a *unique Nash equilibrium* allocation of fishermen:
  - $F(X_A)/X_A = G(X_B)/X_B \Rightarrow 1 = 2 - \hat{X}_B \Rightarrow \hat{X}_B = 1 = \hat{X}_A$.
  - Stable dynamics equalize lake returns in the Nash equilibrium
    - $X_B > 1 \Rightarrow G(X_B)/X_B < 1 = F(X_A)/X_A \Rightarrow$ exit from Lake B.
    - $X_B < 1 \Rightarrow G(X_B)/X_B > 1 = F(X_A)/X_A \Rightarrow$ entry to Lake B.
- Social planner: $\max F(X_A) + G(X_B)$ subject to $X_A + X_B = \bar{X}$
  - FOC equates the social marginal returns: $F'(X_A) = G'(X_B)$.
  - $\Rightarrow$ $1 = 2 - 2X_B^* \Rightarrow X_B^* = 1/2 < 1 = \hat{X}$
  - The lake with diminishing returns is overfished
  - A Pigouvian tax τ^* decentralizes this efficient allocation

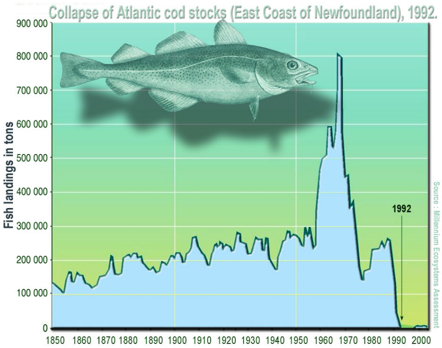
The Tragedy of the Commons

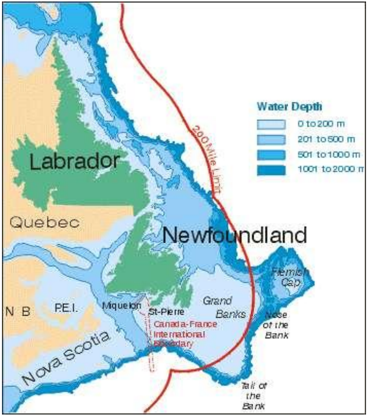


- $G(1/2)/(1/2) - \tau^* = F(1/2)/(1/2) \Rightarrow \tau^* = 1/2$
- Individual decisions are inefficient because they are governed by the social average product and not social marginal product
  - Drivers ignore the slightly increased driving time they inflict on thousands of others

The Fishing Tragedy of the Commons: Newfoundland

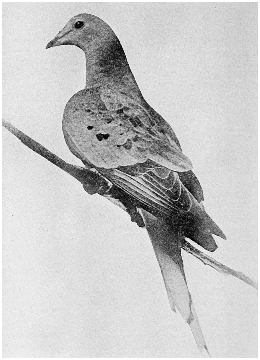
- If there is a stock variable, the tragedy can be permanent

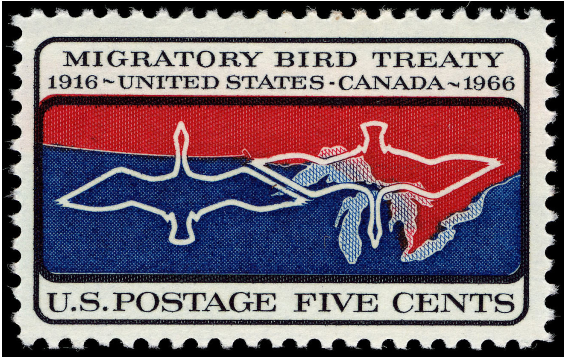




Migratory Birds Tragedy of the Commons Never Happened

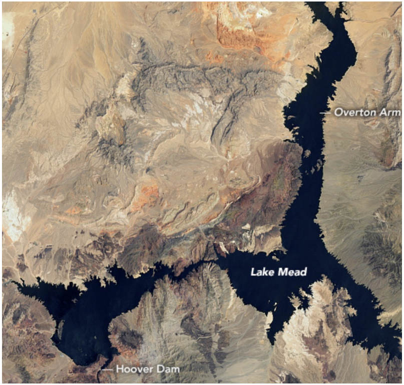
- Martha, the last passenger pigeon, died on September 1, 1914, at the Cincinnati Zoo.
- The Migratory Bird Treaty Act, 1918 banned the possession of migratory birds for commercial purposes
  - Even casting native bird species in movies is against the law!
  - A "feather in your cap" is no longer allowed!





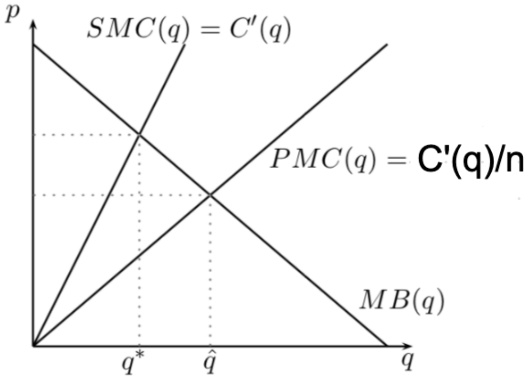
Western Water Tragedy of the Commons

- California is the second-largest rice-growing state in the US!
  - Rice grows submerged in 2" of water during the growing season
- Lake Mead elevation fell from 1,220' (1941) to 1013' (2024)
  - Chinatown (the 1974 Oscar winner) was about water rights



Group Dining Dilemma

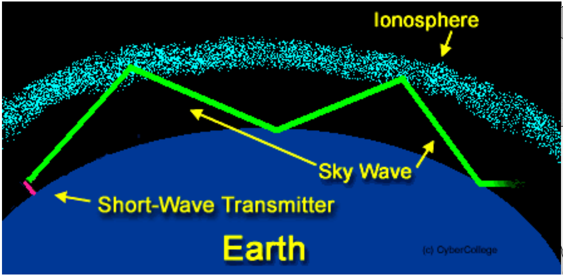
- Assume an agreement or protocol to divide the check equally. can enforce a tragedy of the commons: $SMC > MC$

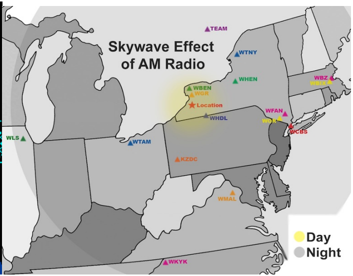


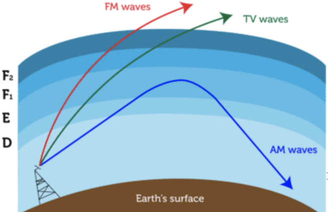
- Everyone then equates $MB = MC$, the *private marginal cost*.
- FOC is $MC = C/n < SMC$, the *social marginal cost*
- Pigouvian tax is "Going Dutch" (paying for their own meal) $\Rightarrow MC = SMC \Rightarrow$ people buy efficient smaller meal $q^* < \hat{q}$.

Examples of Public Goods

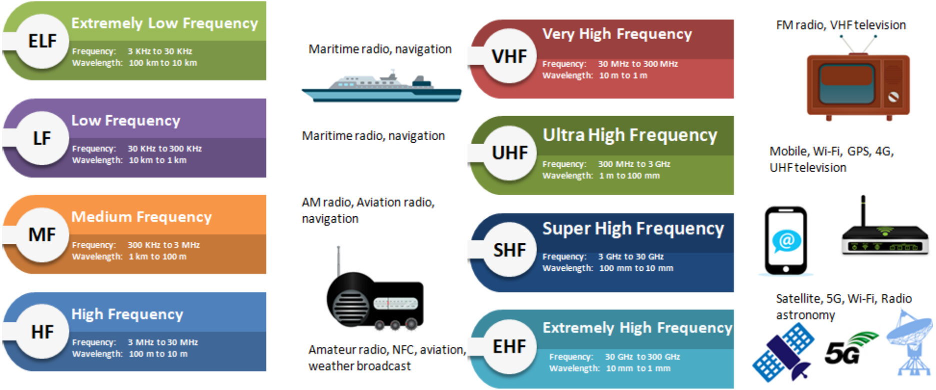
- Information goods (books, music, movies)
  - “He who receives an idea from me, receives instruction himself without lessening mine; as he who lights his taper at mine, receives light without darkening me.” — Thomas Jefferson
- National defense, lighthouses, rural highways, AM/FM radio

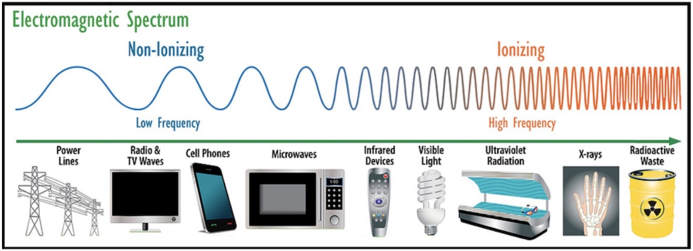






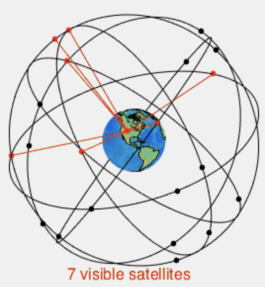
Electromagnetic Spectrum of Public & Club Goods





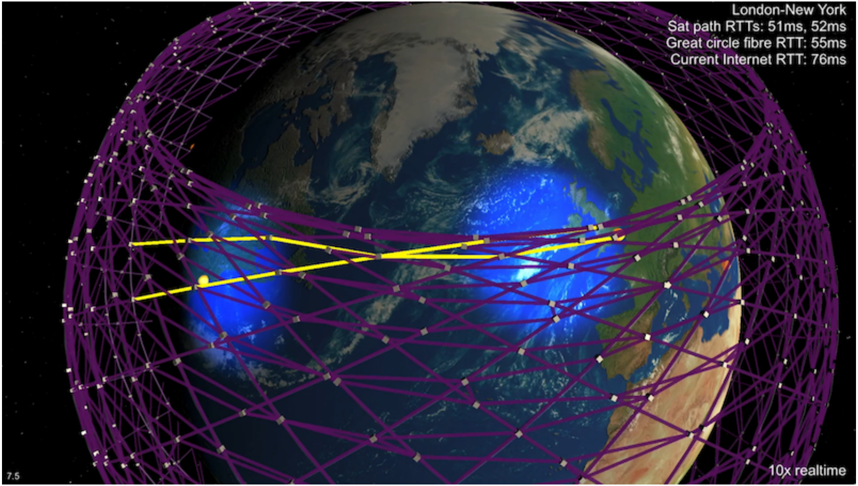
Examples of Club Goods

- Fancy golf courses, toll bridges and roads, satellite radio, etc.
- GPS transitioned from club to public good in May, 2000: government stopped degrading civilian GPS accuracy
- '78: NAVSTAR Global Positioning System satellites launched.
  - They circle the Earth at an altitude of 20,000 km and complete two orbits daily (not in a geostationary orbit)
  - 24 satellites ensure that ≥ 8 satellites can be simultaneously seen at any time from almost anywhere on Earth.



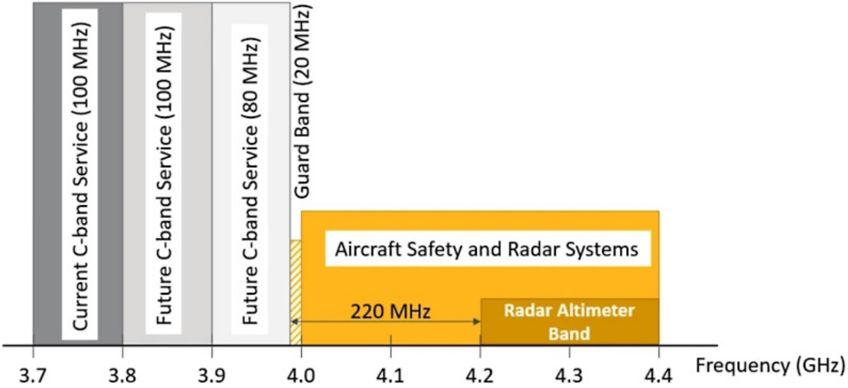
Example of Club Goods: Starlink Satellites

- 3,580 Starlink Satellites, as of today — with 12,000 planned
- Starlink Satellites orbit at 550 km every 95 minutes



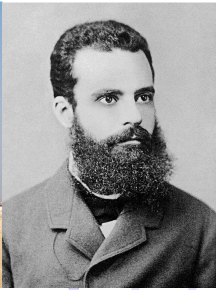
Club Goods in Conflict: 5G vs. Aircraft Altimeters

- Two parties thought they had property rights!



Efficient Provision Nonrival Discrete Public/Club Goods

- Individuals $i = 1, 2, \ldots, n$ have utility $U^i(G, m)$ increasing in amount G of public good and m of private good (money)
- Extensive margin exercise: Build a pyramid $G = 1$ or not $G = 0$
- Pareto Efficiency rule: $G = 1$ if $\exists$ *transfers* $t_1, \ldots, t_n$ from consumers paying for it ($\sum_i t_i \geq c$), such that
  - (a) everyone is weakly better off: $U^i(1, m_i - t_i) \geq U^i(0, m_i)$
  - (b) some j is strictly better off: $U^j(1, m_j - t_j) > U^j(0, m_j)$
- Pareto efficiency is often a very weak social objective
  - Vilfredo Pareto (1848–1923) $\Rightarrow$ fascist? If so, social efficiency



Rear View Mirror on Market Power and Externalities

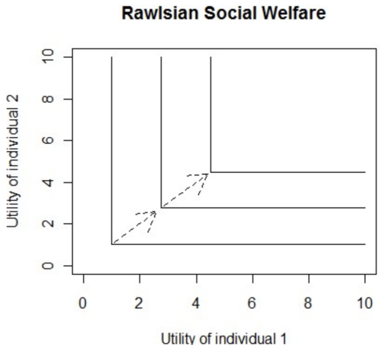
- Congestion public goods: driving, pandemic behavior, etc.
- We analyze these markets as games
  - The Nash equilibria are inefficient since choices reflect average payoffs, not marginal payoffs (as happens with market prices)
  - A Pigouvian tax sometimes restores efficiency
- Binary social choices: use social efficiency as a criterion
- With continuous choices, social efficiency requires a cardinal social objective function! This captures political beliefs.

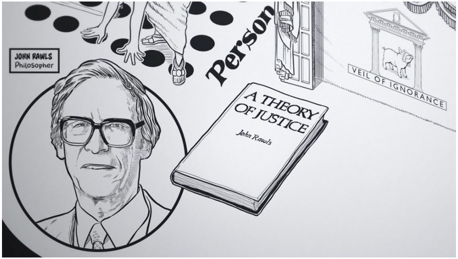
Efficient Provision Nonrival Continuous Public Goods

- We now consider the question of how big to build the pyramid
- *Pareto efficiency is an ordinal social welfare measure*
- We need a cardinal social welfare measure to trade off consumers' gains with an intensive margin
- A social planner, or "society", derives welfare from utilities $u^1, \ldots, u^n$ like a consumer gets utility out of consumed goods
- **social welfare function** (SWF) $W(u^1, \ldots, u^n)$ is increasing and quasi-concave

Rawlsian Social Welfare

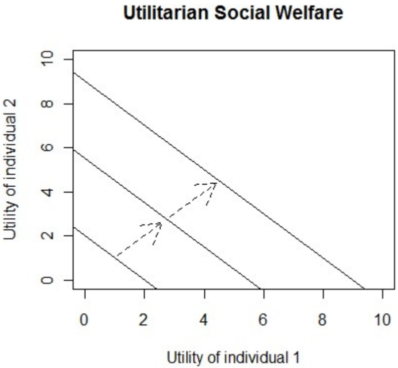
- John Rawls (1921–2002) considered the extreme case of perfect complements SWF: $W(u^1, \ldots, u^n) = \min(u^1, \ldots, u^n)$.



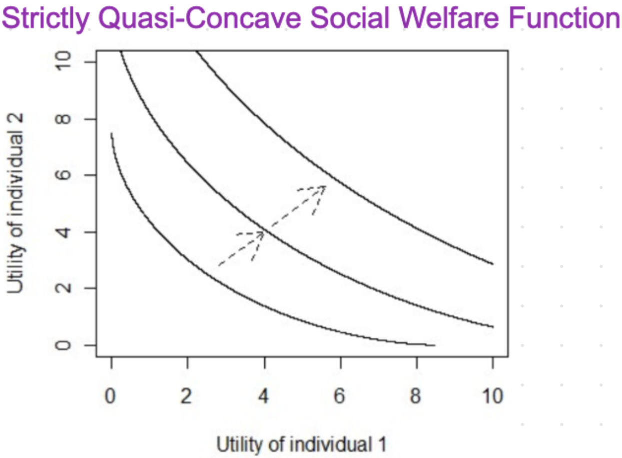


Utilitarian Social Welfare

- Jeremy Bentham (1748–1832): "the greatest happiness of the greatest number is the foundation of morals and legislation"
- Perfect substitutes SWF: $W(u^1, \dots, u^n) = u^1 + \cdots + u^n$



Smooth Strictly Convex Social Welfare

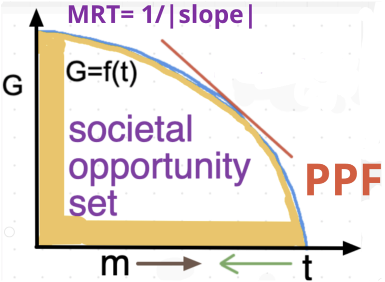


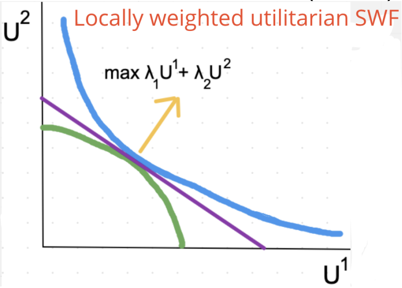
- We will assume this case, with smoothness.

Efficient Provision Nonrival Continuous Public/Club Goods

- Assume just two consumers, paying total transfer $t = t_1 + t_2$.
- Production possibility frontier $G = f(t)$, where $f' > 0 > f''$.
  - PPF is the societal opportunity set in goods space
    - $G = f(t) \Rightarrow t = f^{-1}(G) \equiv T(G) \Rightarrow MC_G = T'(G)$
    - $MRT_{G,m} = T'(G) = \frac{d}{dG} f^{-1}(G) = 1/f'(f^{-1}(G)) = 1/f'(t)$ is inverse |PPF slope| (the marginal rate of transformation)
    - *Always check slope units to verify!*

$\Rightarrow$ Increasing & strictly convex social welfare function $W(u^1, u^2)$





- Don't justify green *utility possibility frontier* (we won't use it)
- $\max_{\{t_1,t_2\}} W(u^1, u^2)$ subject (u^1, u^2) feasible, given the PPF

$\Rightarrow$ Namely, it suffices to use a locally weighted utilitarian SWF

Samuelson (1954), "The Theory of Public Expenditure"

$$\max_{\{t_1,t_2\}} \lambda_1 U^1(f(t_1+t_2), m_1 - t_1) + \lambda_2 U^2(f(t_1+t_2), m_2 - t_2)$$

$$\text{FOC:} \quad \lambda_1 U^1_G f'(t) + \lambda_2 U^2_G f'(t) = \lambda_1 U^1_m = \lambda_2 U^2_m \qquad (\bigstar)$$

- Important but slightly too technical Q: Does the SOC hold?
  - Negishi (1963): *$U(V)$ quasiconcave & $V(X)$ concave* $\Rightarrow$ *$W(X) \equiv U(V(X))$ quasiconcave.* What about the sum? No.
  - Attempt 2: *If h is concave & nondecreasing, and all g_i are concave, then $h = f(g_1, \ldots, g_n)$ is concave.* ($\exists$ 3-line proof)
- Equations ($\bigstar$) uniquely pin down t_1, t_2, and so $G = f(t_1 + t_2)$.
- Divide 1st term in ($\bigstar$) by $\lambda_1 U^1_m f'(t)$ and 2nd by $\lambda_2 U^2_m f'(t)$:

$$MRS_{G,m} \equiv MRS^1_{G,m} + MRS^2_{G,m} = \frac{U^1_G}{U^1_m} + \frac{U^2_G}{U^2_m} = 1/f'(t) = MRT_{G,m}$$

- *Utility weights don't impact public good efficiency condition!*

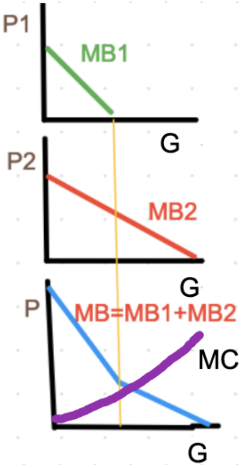
Lemma (The Samuelson Condition, 1954)

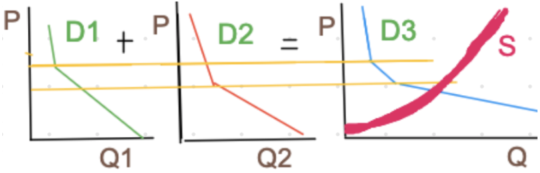
Optimal consumption of public good: $\sum_{i=1}^n MRS^i_{G,w} = MRT_{G,w}$.

- *Quasilinear preferences*: $U^i(G, w) = \phi_i(G) + w$
  - Samuelson's Condition reduces to $\sum_{i=1}^n MB^i(G) = MC(G)$.

Big Picture: Private Goods vs Public Goods Efficiency

- Public goods: common quantity and individual prices
  - ∃ huge identification problem: which consumers are high and low value? (This is solved soon with Lindahl equilibrium)
- Private goods: common price and individual quantities





The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1970

Photo from the Nobel Foundation archive.

Paul A. Samuelson

Prize share: 1/1

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1970 was awarded to Paul A. Samuelson "for the scientific work through which he has developed static and dynamic economic theory and actively contributed to raising the level of analysis in economic science."

- Samuelson's contributions: Revealed preference, public goods theory, geometric discounting, the OLG model, multi sector trade theory, smooth pasting and option pricing, . . .

Peak Load Pricing: an Application of Samuelson Condition

- Different consumers are consumers at different time slots
  - Time period labels resolves the identification problem for Samuelson's solution
- Assume peak and off-peak ferry service to Newfoundland
  - Mid summer is peak ferry time, and off peak is spring and fall
- Price for ferry tickets X (same duration peak and off-peak)
  - Peak season: $p_H = h - X_H$
  - Off peak season: $p_L = \ell - X_L$, where $0 < \ell < h$.
- Costs
  - Ferry *capacity* $\bar{X} \geq \max(X_L, X_H)$ has annual loan cost $\beta > 0$
  - Ferry *operating costs* $b > 0$ to run (crew and fuel).
- Consumer surplus (due to linear demand curves)

$$CS(X_L, X_H) = X_L^2/2 + X_H^2/2$$

- Producer surplus (substituting demand curves)

$$PS(X_L, X_H) = (h - b - X_H)X_H + (\ell - b - X_L)X_L - \beta\bar{X}$$

- The social planner maximizes consumer plus producer surplus

$$CS(X_L, X_H) + PS(X_L, X_H) = (h - \tfrac{1}{2}X_H - b)X_H + (\ell - \tfrac{1}{2}X_L - b)X_L - \beta\bar{X}$$

Social Planner's Solution of Peak Load Pricing

- Lagrangian, where multipliers are shadow prices of capacity:

$$\mathcal{L} = CS(X_L, X_H) + PS(X_L, X_H) + \lambda_H(\bar{X} - X_H) + \lambda_L(\bar{X} - X_L)$$

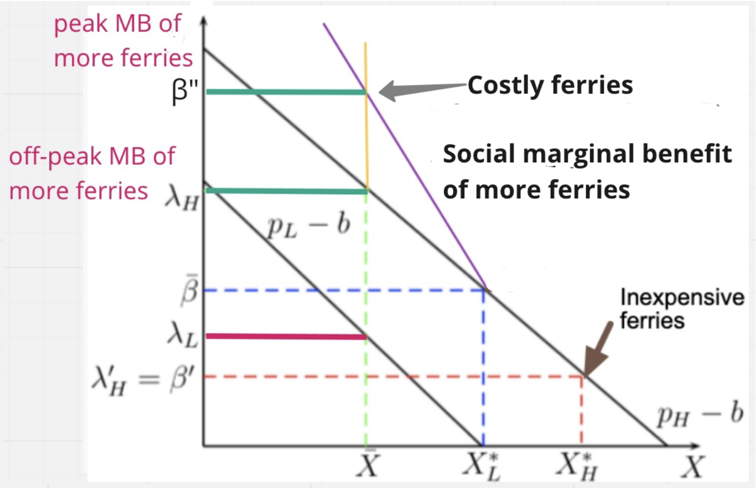
- Kuhn Tucker conditions for $X_L, X_H, \bar{X}, \lambda_H, \lambda_L$, if $X_L, X_H > 0$:

$$\begin{aligned}
[X_H]: &\quad h - X_H - b = \lambda_H \\
[X_L]: &\quad \ell - X_L - b = \lambda_L \\
[\bar{X}]: &\quad \lambda_H + \lambda_L = \beta \\
[\lambda_H]: &\quad X_H \leq \bar{X}, \lambda_H \geq 0, \lambda_H(\bar{X} - X_H) = 0 \\
[\lambda_L]: &\quad X_L \leq \bar{X}, \lambda_L \geq 0, \lambda_L(\bar{X} - X_L) = 0
\end{aligned}$$

- Case by case analysis, depending on which constraint binds
- Case 1: costly ferries, say $\beta > \bar{\beta}$
  - Few are bought, and all run at off-peak times $X_L = X_H = \bar{X}$.
  - Both peak and off-peak pay for the ferries
    - off-peak pays $\lambda_L > 0$ and peak pays $\lambda_H > \lambda_L$
    - $\lambda_L + \lambda_H = \beta$
- Case 2: cheap ferries, say $\beta \leq \bar{\beta}$
  - Many are purchased, and not all are run at off-peak times
  - Peak demand pays all ferry capital costs: $\lambda_L = 0$ and $\lambda_H = \beta$

Social Planner's Solution of Peak Load Pricing Illustrated

- We plot the net marginal value of ferries at peak and off-peak, and the social marginal benefit of ferries (the vertical sum)
- Assume a low & high ferry marginal costs $\beta' < \bar{\beta}$ and $\beta'' > \bar{\beta}$

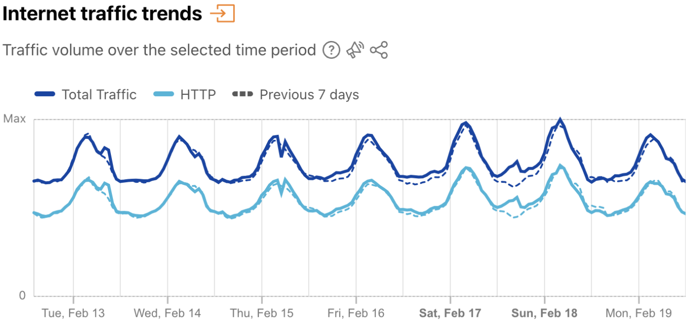


Peak Load Pricing: Container Ship Happens

- Shipping containers arriving from China are emptied of their cargo, reloaded with American goods, and sent back to China.
- Supply crisis changed that!
- 75% containers from the USA to Asia were empty in 2021

Net Neutrality and Peak Load Pricing

- One internet supports peak (night-time) and off-peak periods
- This justifies transfer payments for nighttime internet traffic
- Net neutrality $\Rightarrow$ inefficient breakdown of the web traffic
- Net Neutrality advocates don't want peak load pricing
- Research Q: What would be the social cost of net neutrality?
- Internet traffic peaks midafternoon daily, later weekends



Cleverly Implementing Samuelson's Public Goods Solution

- Lemma: *The public goods competitive equilibrium is inefficient*
  - Pf: Mr i solves $\max t_i(f(t_1+t_2), m_i - t_i)$. So $U^i_G f'(t) = U^i_m$, and so $MC_G = MRS^i_{G,m} < MRS_{G,m}$. So i chooses tax $t_i < t_i^*$.
- Why should we expect Samuelson's outcome to arise?
- **In 1919**, Erik Lindahl decentralized the efficient outcome
- He devised a game (mechanism) whose unique Nash equilibrium is the efficient Samuelson public goods outcome
  - ★ Nash (1950) and Samuelson (1954) came decades later!
- Private good x and public good G, sold at a linear price p
  - Private good endowment $(w_1, \ldots, w_n)$
- Example: How should n roommates pay for a Wi-fi router?
- *A* **Lindahl Equilibrium** *is a public and private goods allocation* $(\hat{G}, \hat{x}_1, \ldots, \hat{x}_n)$, *and individual public good prices* $(p_1, \ldots, p_n)$ *with sum* $p = p_1 + \cdots + p_n$, *such that every consumer* i *chooses* $(\hat{G}, \hat{x}_i)$ *given a price* p_i *for* G*:*

$$(\hat{G}, \hat{x}_i) = \arg\max_{x_i, G} U^i(G, x_i) \quad \text{s.t.} \quad x_i + p_i G = w_i \qquad (\bigstar)$$

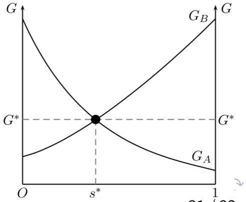
Lindahl Equilibrium

- Namely, knowing that he must pay a share p_i of the price p of the router, every Ms. i agrees on the same public good $\hat{G}$.
- Equilibrium is efficient for cleverly-chosen *personalized prices*

Theorem

A Lindahl Eq'm exists. It is efficient for a strictly convex SWF.

- Intuition: Lindahl Equilibrium asks that individuals pay for the public good according to their marginal benefits
- *Proof*: Assume consumer optimization (★) holds for all i
- FOC $\Rightarrow p_i = U^i_G / U^i_x = MRS^i_{Gx}$ for all $i = 1, \dots, n$
- Define the price $p = \sum_i p_i$.
  - $\Rightarrow \sum_i MRS^i_{Gx} = p_1 + \cdots + p_n = p = MRT_{Gx}$
  - $\Rightarrow$ Samuelson public goods efficiency condition holds $\Rightarrow G = G^*$
- Proof depiction for roommates $i = A, B$.
  - Let G^* be optimal, and p^* the social MRS
  - G^* maximizes $U^i(G, x_i)$ s.t. $x_i + s_i pG = w$.
  - Personal prices $(p_A, p_B) = (s^* p, (1 - s^*)p)$ yield a Lindahl equilibrium if:
- Strictly convex preferences $\Rightarrow$ falling demands.



Cobb Douglas Preference Lindahl WiFi Router Example

- Assume $U^A(G,x) = x^{1-\alpha}G^{\alpha}$ and $U^B(G,x) = x^{1-\beta}G^{\beta}$.
- Cobb Douglas: $G_A^* = \alpha w_A/(sp)$ and $G_B^* = \beta w_B/[(1-s)p]$.
- Finally, agreement on router size $G_A = G_B$ implies:

$$\frac{\alpha w_A}{sp} = \frac{\beta w_B}{(1-s)p} \quad \Rightarrow \quad s^* = \frac{\alpha w_A}{\alpha w_A + \beta w_B}$$

- *A* pays more for the more he likes Wifi and the wealthier he is.
- Private goods
  - *Different people can consume different quantities, but in competitive equilibrium, they all pay the same prices.*
- Public goods
  - *Everyone consumes the same amount quantity, but in Lindahl equilibrium, different people may pay different prices.*