An Economic Theory Masterclass

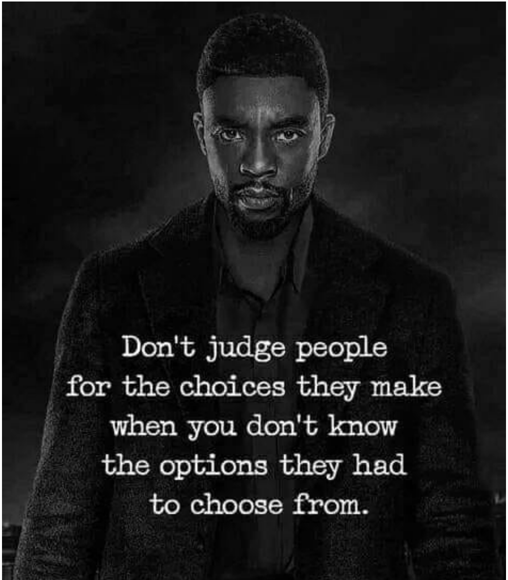
Part II: Competitive Markets in Partial Equilibrium

Lones Smith

February 28, 2024

Paul Samuelson Produced this Economic Idea

- And not Chadwick Boseman



Rear View Mirror on Matching (TU)

- Allowing for transfers, efficiency becomes an equal treatment measure of social goodness ("better" is well-defined)
  - A unique stable matching need not be efficient
    - E.g. because comonotonicity $\neq$ SPM (musician matching)
- Competitive equilibrium: everyone's paid $\geq$ best outside option
  $\Rightarrow$ many incentive constraints (not unique?)
    - 713B topic: Auction theory integrates constraints, proving all auctions give the same revenue (Revenue Equivalence Th'm)
- Welfare Theorems
  A. Competitive equilibrium is efficient: easy contradiction proof
  B. Efficiency can emerge in a competitive equilibrium
    - Proof: LP duality (primal = dual) yields multipliers on constraints; these shadow values act as competitive prices
    - The dual is less complex to compute
    - Shadow values may be:

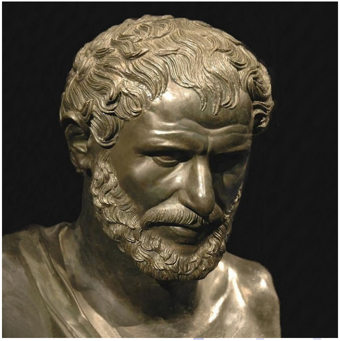
  Eg. 1. wages in the employment model
  Eg. 2. consumer and producer surplus in the trading model
  Eg. 3. payoffs and rents in the location assignment model
- Becker Marriage: PAM/NAM $\Leftrightarrow$ SPM/SBM (extreme cases!)
  - Trade surplus is SBM $\Rightarrow$ NAM matching in a double auction

Supply and Demand

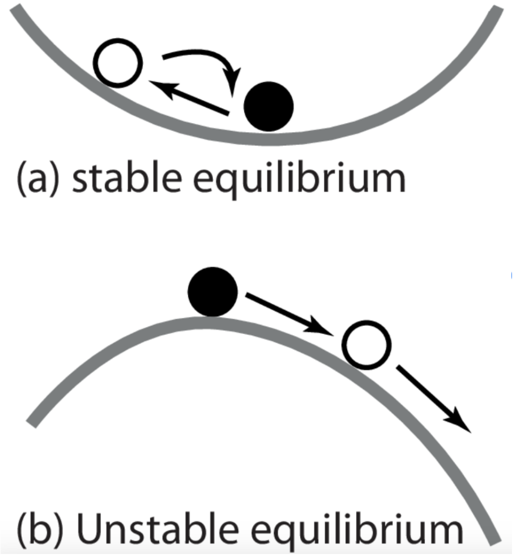
- Assume a competitive price-taking environment
- Double auctions: just an extensive margin (in or out) for all trades
  - WTP (willingness to pay) and WTA (willingness to accept)
- Supply & demand curves will also reflect intensive margins
- usually upward sloping supply curve
- usually downward sloping demand curve
  - very negative income effects $\Rightarrow$ demand rises in price
  - addictive behavior $\Rightarrow$ WTP rises with quantity (oh no, drugs)
- These two curves answer out-of-equilibrium hypothetical "what if" questions: what would the supply and demand be at any other price?
- By parsing our logic into supply and demand, we can compartmentalize our analysis, and make clearer predictions
  - Supply and Demand: "Father Guido Sarducci's 5 Minute University"

Ours "Static" Models are Really Steady States

- Supply quantity Q^S and inverse supply price P^S
- Demand quantity Q^D and inverse demand price P^D
- The model need not be static. *Everything could be steady-state!*
  - Supply and demand could be flows (units are per week, or per day)
  - Life is all about dynamics: Heraclitus — Panta Rhei
    - "All entities move and nothing remains still"
  - "No man ever steps in the same river twice"



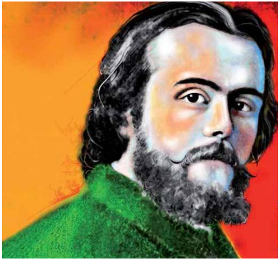
Stability



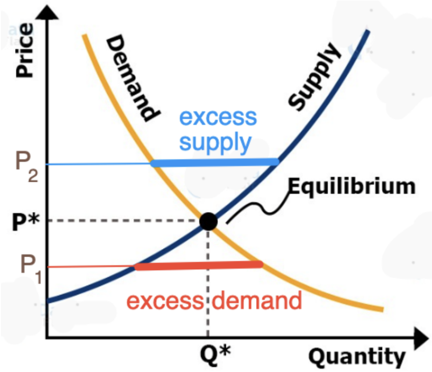
- Unstable equilibria are not reliable fixed points

Stability: Does Competitive Equilibrium Happen?

- Why does market equilibrium arise?
- adjustment *tatonnement process* — check Google translate :)
- Walrasian price stability (*Elements of Pure Economics*, 1874)
  - price adjustment process of fictional double auctioneer
  - $\Rightarrow$ change in the price shares the sign of *net demand* $Q^D(P) - Q^S(P)$.



Walrasian Stability

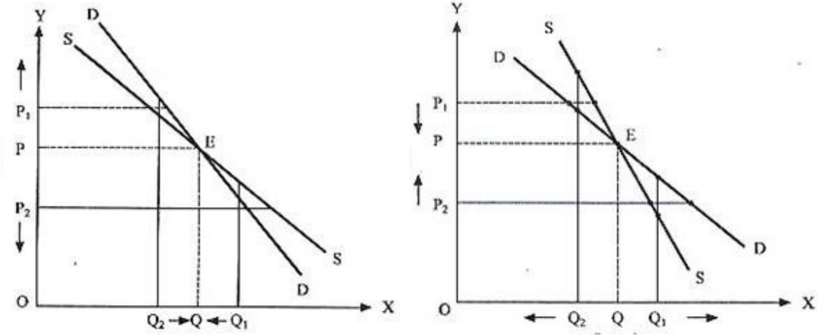


- Dynamic stories
  - Search by people who engage in pairwise bargaining over prices
  - forward-looking optimization about willingness to accept
  - *During the adjustment, the short side of the market fixes quantity.*
    - Demanders won't demand more than they want at that price.
    - Suppliers won't sell more than they are willing at that price.

Detour: The Market "Learns"

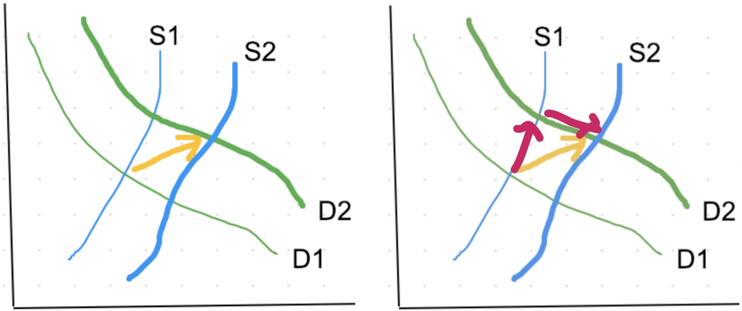
- The market is the ultimate in artificial intelligence
- Groups of individuals might screw up but the larger market learns
- Financial Crisis of 2008: When markets do not learn, we are stunned
  - How could the price not clear the market?
  - The answer is that our story misses something about "money"
  - The IOU nature of money created a game of strategic complements — which tend to have multiple equilibria
  - Advanced Theory Topic: Games of Strategic Complements

Stability: Downward-sloping Demand and Supply



- Supply steeper than demand $\Rightarrow$ Walrasian stable
- Demand steeper than supply $\Rightarrow$ Walrasian unstable
- So Walrasian stability holds iff $Q^S_P(P) > Q^D_P(P)$
  - ...formulated using direct and not inverse supply & demand curves!
- Not Even A Thinker Q: What if supply and demand slope up?

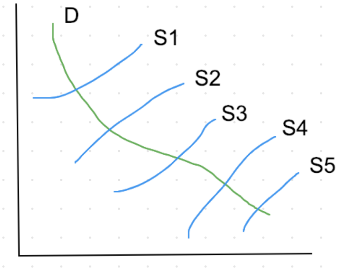
Comparative Statics aka Comparison of Steady States Analysis



- Comparative statics are a peasant's comparative dynamics
- Intuitively, monotone dynamics from one steady-state to the next $\Rightarrow$ comparing the two static situations is informative of dynamics
- What if demand shifts quickly, but supply shifts slowly?

Identification of Supply and Demand Curves

- Price and quantity reflect both supply and demand.
- If you wanted to "identify" the demand curve, you find something that just shifts supply and leaves demand invariant.
- Ragnar Frisch (1933) first highlighted the identification problem — first winner of Economics Nobel prize (1969)
- With enough variation in supply, we can identify the demand.
- Likewise, variation in demand but not supply would allow one to pin down the supply curve.



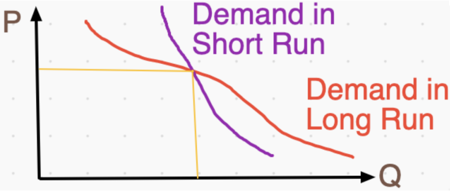
Deja Vu: Flash Elasticities Review of Economics 711

- For small price changes:

$$\mathcal{E}(Q,P) = \frac{dQ}{dP}\frac{P}{Q} = \frac{d\log Q}{d\log P} \approx \frac{\%change\ quantity}{\%changeprice}$$

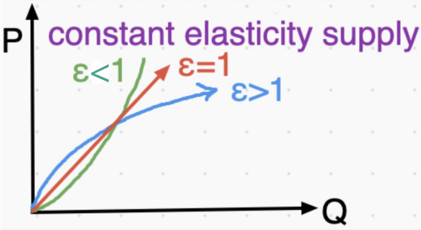
  $\Rightarrow$ Coefficients in log regressions are elasticities
  - Elasticity is a ratio of proportionate changes $\Rightarrow$ unit-free!
  - More elastic supply or demand $\Rightarrow$ quantity changes more if price falls
- The long run has fewer constraints than the short run
- Le Chatelier's Principle: *The absolute change of any choice variable is weakly higher in the longrun than shortrun.*

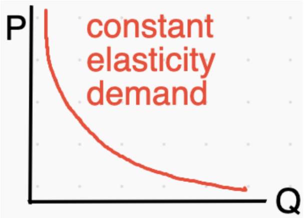
  $\Rightarrow$ |long run elasticity| > |short run elasticity|



Constant Elasticity Supply and Demand Curves

- Let's write the supply or demand curve as $Q(P)$
- Rewrite $Q'(P)P/Q = \varepsilon$ as $dQ/Q = \varepsilon dP/P$
- Integrating yields $\Rightarrow \log Q = \varepsilon \log P + \log K \Rightarrow Q = KP^{\varepsilon}$.
- Hyperbolic downward sloping curves $\varepsilon < 0$: $P \propto Q^{1/\varepsilon}$
- Geometric upward sloping supply curves $(\eta > 0)$ are linear if $\eta = 1$

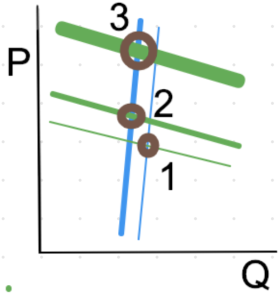




- Supply is elastic if $\eta > 1$ and demand is elastic if $|\varepsilon| > 1$
  - $\Rightarrow$ Quantity changes proportionately more than price
  - PS Demand elasticity is spoken of in absolute terms!

Large Price Volatility in the Oil Market

- Consider the facts of the oil or gasoline market
  - Huge price volatility
  - Minimal quantity volatility
  - Small change in fundamentals (i.e. small shift in supply and demand)



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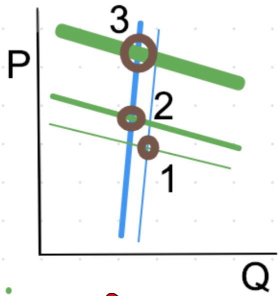

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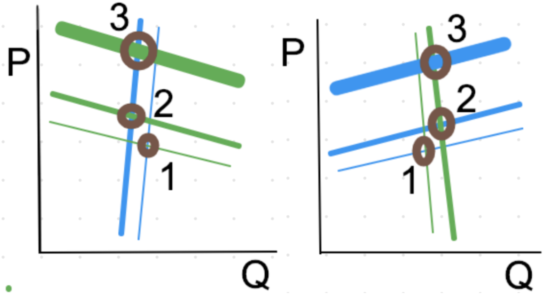

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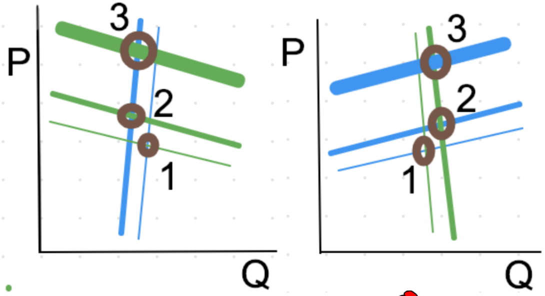

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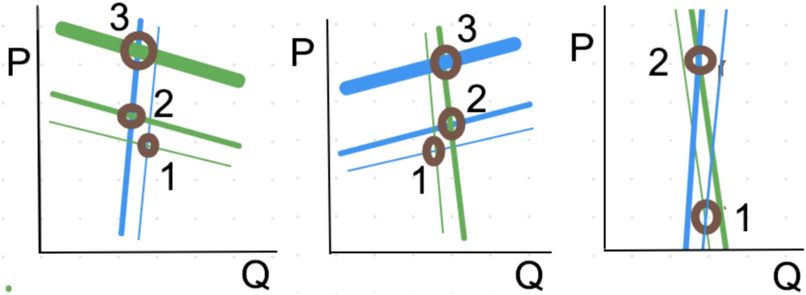

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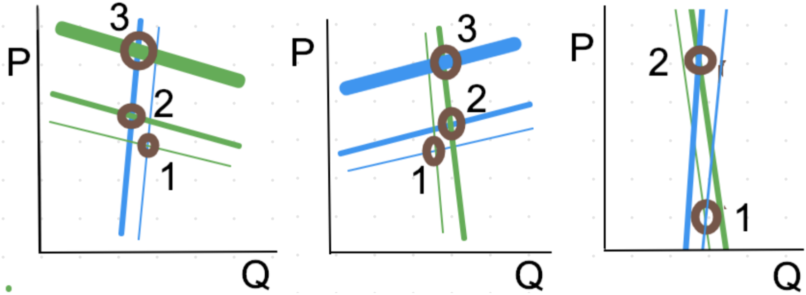

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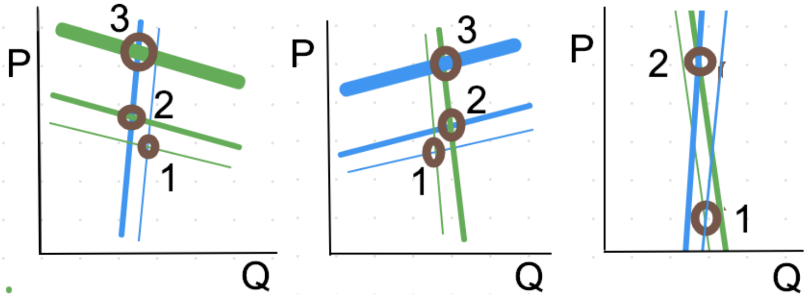
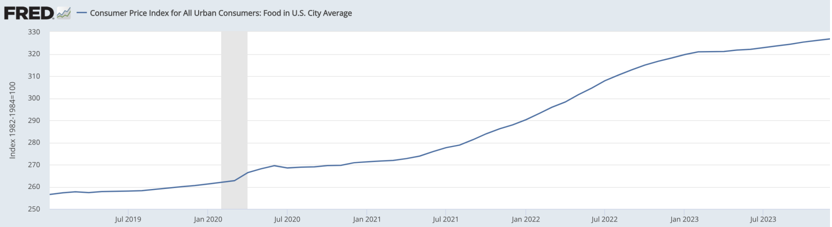


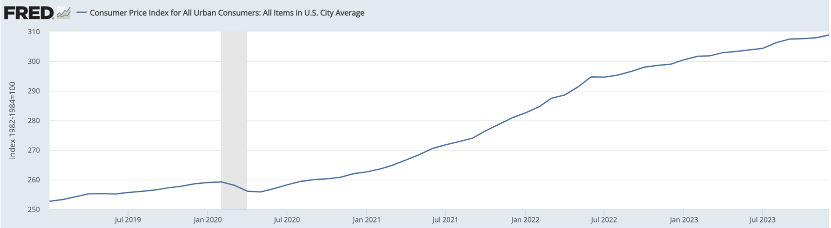
- Small fundamentals shifts cause large proportionate price changes iff both supply and demand are both highly inelastic.
- Inelastic supply or demand $\Rightarrow$ low quantity volatility
- Small fundamentals changes can lead to large quantity changes iff supply and demand are both highly elastic.
- Elastic supply or demand $\Rightarrow$ low price volatility
- Volatility of prices is greater in the short run, of quantity in long run

Thinker: 2020-24 Food Inflation > Average Inflation

- Assume COVID Stimulus Checks Raised Demand
- Food in Cities (24.7% Inflation)

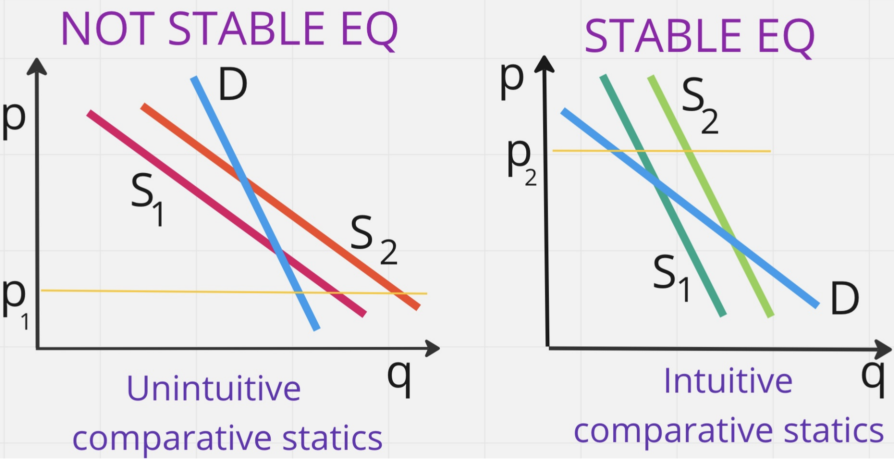


- All Urban Goods (19.3% Inflation)



Samuelson's Correspondence Principle (1941)

- Comparative statics are "intuitive" if the equilibrium is stable: price falls if supply rises, or demand falls
- Standard case: increasing supply and decreasing demand
- More subtle cases: direct supply curve is steeper than demand



This Comparative Statics Slide is Ironically Timeless

- Add a shift parameter to supply $Q^S(P,\beta)$, with $Q^S_\beta(P,\beta) > 0$
- Competitive equilibrium price & quantity solve: $Q^D(P) = Q^S(P,\beta)$
- Implicitly differentiate equilibrium identity in β, with $P(\beta)$ a function:

$$\frac{dP}{d\beta} = \frac{-Q^S_\beta(P,\beta)}{Q^S_P(P,\beta) - Q^D_P(P)} \qquad (\bigstar)$$

$\Rightarrow$ Price falls when supply rises, provided stable: $Q^S_P(P,\beta) > Q^D_P(P)$

- Multiply $(\bigstar)$ by $(\beta/P) = (\beta/Q)/(P/Q)$. Then the equilibrium price elasticity is

$$\mathcal{E}(P|\beta) \equiv \frac{dP}{d\beta}\frac{\beta}{P} = \frac{-\mathcal{E}(Q^S,\beta)}{\eta - \varepsilon}$$

- Likewise, let index demand as $Q^D(P,\alpha)$, with $Q^D_\alpha(P,\alpha) > 0$.
- Price rises if demand increases, given a stable equilibrium. Indeed:

$$\frac{dP}{d\alpha} = \frac{Q^D_\alpha(P,\alpha)}{Q^S_P(P,\beta) - Q^D_P(P,\alpha)} = \frac{\mathcal{E}(Q^D,\alpha)}{\eta - \varepsilon}$$

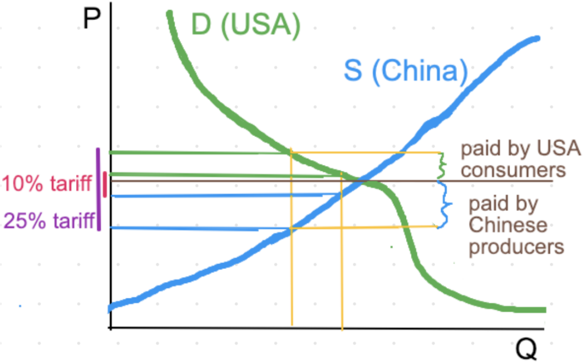
- Home work: Do the quantity comparative statics

Shared Incidence or Tax or Tariff

- Trump added a 10% tariff on Chinese imports, to rise to 25%

⇒ wedge between supply and demand prices: $P_D > P_S$.

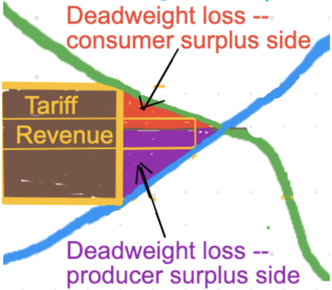
- Incidence: Who pays the tariff or tax?
- "China is paying us billions of dollars in tariffs." — Trump
- Fact: The more elastic is demand, the less of the tariff buyers pay.
- Fact: The more elastic is supply, the less of the tariff suppliers pay.



Deadweight Loss of Tax

- Double auctions: No effect of small tax! Here: small effect.
- Lost gains from trade = lost consumer + producer surplus
- Assume tariff revenue is socially neutral: gain to government balances loss to producers or consumers

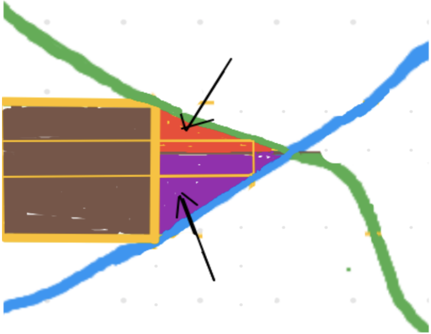
⇒ deadweight loss (excess burden) of tariff is red + purple

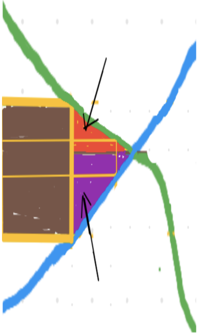


←Taxes erase marginal trades

Changes in the Deadweight Loss of Tax

- The deadweight loss of a tariff increases in the quantity reduction, larger with more elastic demand or supply





(less elastic S and D)
(shortrun)

Tax Irrelevance Theorem

- Tariff or sales or *ad valorem* tax: $P_D(Q) = P_S(Q) + \tau P_S(Q)$
- *Specific tax* τ: $P_D(Q) = P_S(Q) + \tau$
  - Wisconsin specific tax examples
    - Gas tax: state 32.9¢ and federal 18.4¢ per gallon
    - Beer: 6¢/gallon and wine: 25¢/gallon and liquor: $3.25/gallon
    - Also exists for cigarettes
- Specific tax is easier to analyze: parallel demand / supply shift

Theorem (Tax Irrelevance Theorem)

Regardless of whether demand or supply pays a specific tax, the demand and supply prices, market quantity, and efficiency loss are the same.

- USA: A sales tax is paid by demanders ⇒ down-shift in demand
- Most of world: VAT (hidden tax) is paid by suppliers ⇒ up-shift in supply, since the marginal cost of sellers is higher by the tax

Elasticities and Tax Incidence: Who pays the tax?

- A small tax has no effect in a double auction.
- In our continuous world, we focus on a small tax (Taylor series)
- The more inelastic side of the market pays more of a tax and benefits more from a subsidy, but how much more?
  - Demand elasticity $\varepsilon = D'(P)(P_D/Q_D) < 0$
  - Supply elasticity $\eta = (dQ_S/dP_S)(P_S/Q_S) > 0$

Theorem (Tax Incidence Theorem)

The share of a small tax τ paid by demand is $\frac{\eta}{\eta-\varepsilon}$, and by supply is $\frac{-\varepsilon}{\eta-\varepsilon}$.

- *Proof*: By Tax Irrelevance Theorem, impose the tax τ on demand.
- Differentiate $D(P(\tau)+\tau) \equiv S(P(\tau))$, where $P(\tau)$ is supply price
- Hence, $D'(P(\tau)+\tau)(P'(\tau)+1) = S'(P)P'(\tau)$
- Supply price slope in the tax:

  $$\Rightarrow \qquad P'(\tau) = \frac{D'(P(\tau)+\tau)}{S'(P)-D'(P(\tau)+\tau)} \approx \frac{\varepsilon}{\eta-\varepsilon} \in (-1,0)$$
- Finally, demand price rises with slope $P'(\tau)+1 \approx \eta/(\eta-\varepsilon) \in (0,1)$

Deadweight Loss for Small Taxes

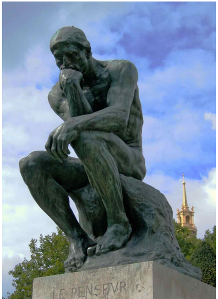
- Since $\varepsilon = D'(P)(P/D)$, the quantity demanded changes by

$$dQ = \epsilon \frac{QdP^D}{P^D} \approx \epsilon \left(\frac{\eta}{\eta - \epsilon} \right) \tau \left(\frac{Q}{P^D} \right) = \left(\frac{1}{\frac{1}{\epsilon} - \frac{1}{\eta}} \right) \tau \left(\frac{D}{P^D} \right)$$

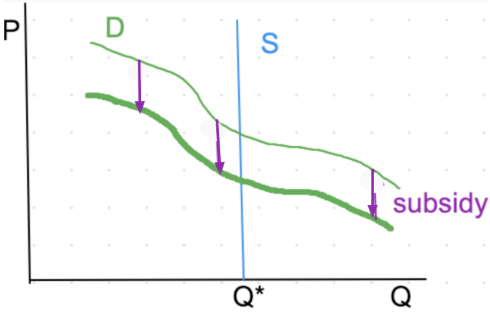
- Deadweight loss: Lost gains from trade = lost CS + PS
- Hence, the deadweight loss is the area of the standard triangle:

$$\frac{1}{2}(dQ)(dP^D - dP^S) = \frac{1}{2}(dQ)\tau \approx \left(\frac{1}{\frac{1}{\epsilon} - \frac{1}{\eta}} \right) \left(\frac{Q}{2P^D} \right) \tau^2$$

- Exercise: check the units in this formula!
- Thinker: What about Quantity Taxes?
  - Feudal system: Give a tithe of crops to the church!
  - *Tithe* τ: $P^D(Q) = P^S(Q + \tau)$



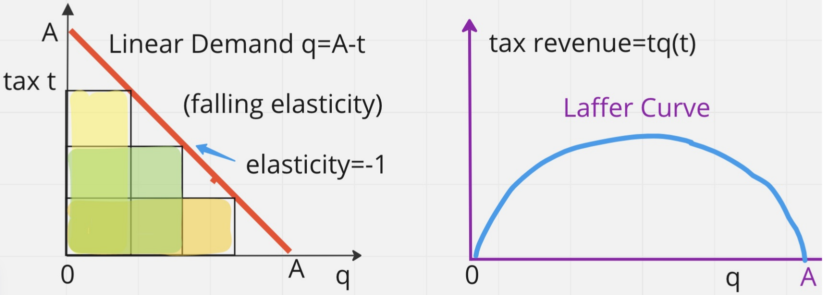
Political Economy of Taxes: Tax or Subsidy Incidence



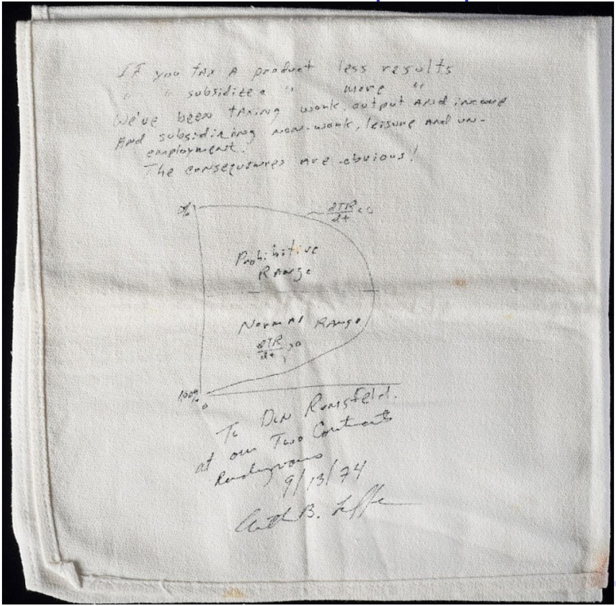
- Tax or subsidy incidence invariably explains who pushes for it
- In 2009, Michigan ended the Promise Scholarship program, giving 96,000 in-state students up to \$4,000 for college
  - Can't ↑ shift supply curve ⇒ shift demand (Tax Irrelevance Theorem)
  - Who fought to keep the subsidy? Colleges! (Tax Incidence Theorem)
- Take our message for governments: taxing inelastic supply is efficient

Demand Elasticity and the Laffer Curve for Total Revenue

- Tax revenue $tq(t)$ is rising / falling when $tq'(t) + q(t) \gtrless 0$ iff $\varepsilon \gtrless -1$
- If tax revenue peaks at an intermediate quantity, then this rules out a constant elasticity demand
- Linear demand curves have falling elasticities $|\varepsilon| = \left|\frac{dq}{dp}\frac{p}{q}\right| = p/q$
- Tax revenue is maximized (in example midway, as slope is minus one)



Art Laffer's 1974 Back of the Envelope Explanation to Rumsfeld



Public Finance: the Ramsey Inverse Elasticity Tax Rule

- Social planners hate deadweight losses
  - $\Rightarrow$ Optimal taxes minimize deadweight losses for any given revenue
- Tax revenue falls when the tax rises if the demand is elastic:

$$[D(P+\tau)\tau]' = D(P+\tau) + D'(P+\tau)\tau = D(P+\tau)[1+\varepsilon\tfrac{\tau}{P+\tau}]$$

  $\Rightarrow$ never tax an elastically demanded good
- Ramsey (1927): Minimize the social cost of raising revenue R

$$\max V(p+\tau, I) \text{ s.t. } \tau \cdot x(p+\tau, I) \geq R$$

  where $V(p, I)$ is the indirect utility function for prices p and income I
  - Cool! This long predates the 1950 invention of Kuhn Tucker analysis!!
- **Ramsey inverse elasticity rule**:
  "taxes should be proportional to the sum of the reciprocals of its supply and demand elasticities"
- $\Rightarrow$ governments shouldn't tax elastically demanded goods or supplied goods

Planner Optimization SOC Story for Stability (Lones' Lemma)

- Maximize $U(x, \beta)$, a twice differentiable function.
- What is $x'(\beta)$?
  - FOC $U_x(x, \beta) = 0$ at an interior solution.
  - Differentiate FOC $U_{xx}(x, \beta)x'(\beta) + U_{x\beta}(x, \beta) = 0$.
  - Use SOC $U_{xx}(x, \beta) \leq 0$ to get

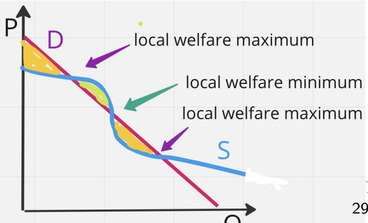
$$x'(\beta) = -\frac{U_{x\beta}(x, \beta)}{U_{xx}(x, \beta)} \propto U_{x\beta}(x, \beta)$$

- Equilibrium comparative statics. What is $p'(\beta)$?
- Lemma: *If demand & supply slope down, welfare* $= \int_p^\infty D(z) - S(z)dz$
  - Proof: Plot the picture — visually, this is integrating by parts.
- Maximize welfare $\int_p^\infty D(z) - S(z)dz$ at competitive equilibrium
  - FOC $D(p) - S(p, \beta) = 0$
  - Use SOC $D_p(p) - S_p(p, \beta) \leq 0$

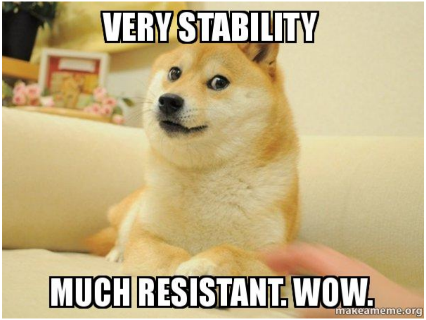
$$p'(\beta) = \frac{-S_\beta(P, \beta)}{S_p(p, \beta) - D_p(p, \alpha)} \propto -S_\beta(p, \beta)$$

  - $\boxed{\text{Stability} \Leftrightarrow \text{SOC of planner!}}$

$\Rightarrow$ Stable equilibrium is a local welfare max



Rear View Mirror on Competitive Supply and Demand



- Demand curve fall & supply curves rise $\Leftrightarrow$ heterogeneity & convexity
- Both P and Q change given shocks — Q more with greater elasticity
- Stability $\Leftrightarrow$ signed elasticities $\eta > \varepsilon$
- Correspondence Principle: stability $\Rightarrow$ intuitive comparative statics
- Less elastic side of market pays more of a tax (political economy 101)
  - Laffer curve. PS Optimal taxation says tax more elastic goods less
- Utilitarian social welfare: area between S & D curves (units $(\frac{\$}{q})\times q = \$$)
- Planner's SOC $\Leftrightarrow$ stability of equilibrium

Optimal Taxation Theory Explains Real World Taxes

- Ramsey's basic insight is intuitively understood by governments
- They know to tax inelastically supplied resources:
  - Oil taxes, mineral taxes
  - existence tax: poll tax (head tax) in Britain (fertility impact?)
  - wealth taxes are usually real estate, or at death taxes
  - millionaire tax? billionaire tax?
- More rationality $\leftrightsquigarrow$ more elastic response
  - Example: Does income reflect effort, ability, luck or networks?
  - Tax luck or ability or networks — inelastically supplied. Politically:
    - left wing thinks earnings reflect luck & networks more, right wing effort
    - left wing understates elasticities $\Rightarrow$ higher peak of Laffer curve
- Funny example of a tax fail:
  - 2008, Maryland "millionaire's tax" of 6.25% tax on income $> \$1M$
    - 30% drop in millionaire's taxpayers and 22% drop in declared income.
    - $\Rightarrow$ income taxes from this group fell by \$257 million
    - Tax ended in 2010

Supply / Demand Curves: Intensive and Extensive Margins

- We introduced the supply and demand in the double auction
- There, all gains from trade — namely, producer plus consumer surplus — reflect heterogeneity.
- We now allow a realistic intensive margin,
  - Output from every firm, and consumption from every consumer, increases in the market price
  - the producer surplus also increases in cost convexity, and consumer surplus increases in preference convexity

Deja Vu: Flash Cost Function Review of Economics 711

- **Escapable** costs can be avoided vs. **sunk** (inescapable) costs
  - "Sunk costs are sunk": they cannot possibly affect dynamically rational behavior, and should be ignored
  - = essence of dynamic programming
- A **fixed cost** is invariant to the quantity.
  - It can be sunk or escapable
- A **variable cost** has an intensive margin
  - So variable costs are escapable (just vary them down to zero)
  - Marginal costs are the derivative of variable costs
  - Average costs are fixed plus variable costs divided by quantity
- Optimization Big Picture
  - All firms equate marginal costs and price $\Leftrightarrow$ intensive margin
  - All firms: Average costs $\leq$ price $\Leftrightarrow$ extensive margin (no exit)
  - Marginal firm: Average costs $=$ price $\Leftrightarrow$ extensive margin (no entry)

Deja Vu: Short, Medium, Long Runs Review of Economics 711

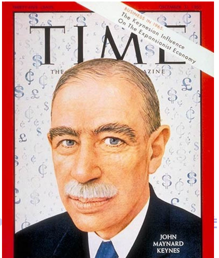
- As the run increases, there are more choice margins, and so inescapable costs $\rightsquigarrow$ escapable (e.g., rental contracts end).
- **Short run**
  1. fixed costs are inescapable; cost function is just variable costs
  2. Insufficient time for entry; reducing output to zero
     - *Ukraine consumes entire UK supply of artillery every 8 days!*
- **Long run**
  1. All costs are escapable, and so are included in the cost function

  $\Rightarrow$ *Costs are higher in the long run than the short and medium runs*

  2. firms enter if there are profits to be made and otherwise exit
     - John Maynard Keynes: "In the long run we are all dead"
     - Naturally, Keynes developed a short run theory
- **"Medium run"**
  - more decision margins available

    $\Rightarrow$ more costs escapable than in short run

    $\Rightarrow$ fewer costs escapable than in long run
- Time Magazine Cover 12/31/1965 $\longrightarrow$



Long Run Supply with Homogeneous Firms and Intensive Supply

- Goal: show how intensive and extensive margins interact
- We explore an illustrative extended example, focusing on supply!
- **Industry supply curve** locus (Q, P)
  - Taking P as given, existing firms i in the short run, or all potential firms in the long run — profitably produce q_i, and $Q = q_1 + \cdots + q_n$
  - Price-taking behavior is incredible with few firms
- Cost functions $C(q) = 1 + q^2$ (fixed cost 1 & variable cost q^2)
- Continuous quantity allows us to compute supply by differentiation!
- Optimal production: $C'(q) = P \Rightarrow$ output $q^* = P/2$.
- Long Run
  - No firm wishes to enter or exit, with all costs escapable: $P = C(q)/q$

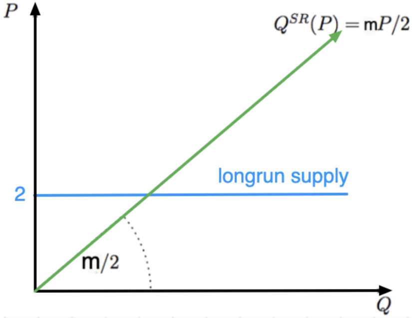
$$2q^* = C'(q^*) = P = C(q^*)/q^* = \frac{1}{q^*} + q^* \Rightarrow 2q^* = \frac{1}{q^*} + q^* \Rightarrow q^* = 1 \Rightarrow P = 2$$

$\Rightarrow$ The long run inverse supply curve is $P = 2$.

- *Every firm earns zero profits in the long run*

Short Run Supply with Homogeneous Firms and Intensive Supply

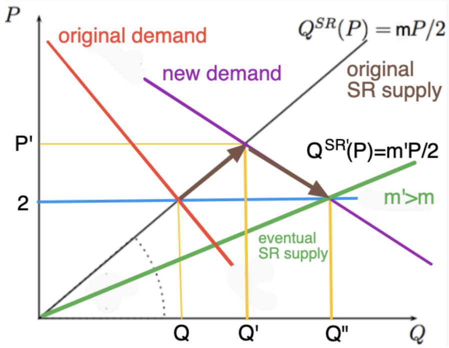
- Short run: each firm still produces $C'(q) = P \Rightarrow$ output $q^* = P/2$
  - This intensive margin effect — firms sell more with a higher price — was absent with double auctions
- Fix the mass m of firms $\Rightarrow Q_m^{SR}(P) = mq = mP/2$ (😱)



- All firms earns positive profits: $C_{SR}(q) = (q^*)^2 \Rightarrow AC = q^* < P$
- The short run supply curve rises simply due to cost convexity.
  - Short run profits owe to cost convexity (diminishing returns is good?)
  - Example: The same firms produce, but use overtime

Short Run and Long Run Response to a Demand Increase

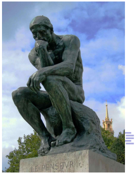
- Short run
  - Every firm produces more (along its marginal cost curve)
  - The price increases to $P' > 2$ and the quantity to $Q' = Q^{SR}(P') > Q$
  - Quasi-rents: temporary positive profits during adjustment ($AC < P$)
- Long run (after enough time passes so that entry occurs)
  - Firm mass rises to $m' > m$ so that short run supply allows $P = 2$
  - $\Rightarrow$ quantity rises to $Q'' > Q'$
  - Entry $\Rightarrow$ long run supply is more elastic (Le Chetalier's Principle)



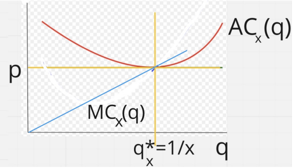
Supply with Heterogeneous Firms and Intensive Supply

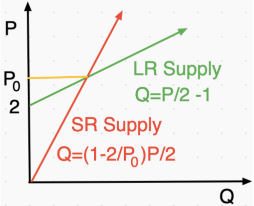
- Firm with index x has costs $C_x(q) = 1 + x^2 q^2$
- Assume the index x has a unit mass density on $[1, \infty)$
- Higher index firms produce less output q_x when positive
  - Firm x supplies $2x^2 q_x = MC_x = P \Rightarrow$ supply $\boxed{q_x(P) = P/(2x^2)}$ (😬)
- Short run: No one shuts down, since the price exceeds non-sunk costs: $AC_x(q) = x^2 q_x < 2x^2 q_x = MC_x(q) = P$
- Long run
  - The fixed cost 1 is escapable, and included in costs

  $\Rightarrow$ $AC_x(q) = 1/q_x + x^2 q_x = 2x^2/P + P/2 \leq P$ for all firms $x \leq \frac{1}{2}P$
    - $\Rightarrow$ U-shaped average costs
    - $\Rightarrow$ *minimum efficient scale* of firm x is $q_x^* = 1/x < 1$.
    - $\Rightarrow$ The minimum average cost is $AC_x(q_x^*) = 1/q_x + x^2 q_x^* = 2x \geq 2$
    - *Marginal firm earns 0 profits at min AC*: $P = AC_x(q_x^*) = 2x$
    - Why? The min AC is the most efficient a firm can be!
    - $\Rightarrow$ Marginal firm is $x(P) = \frac{1}{2}P$
  - Price ≥ 2: must pay for minimum average costs
- Thinker: Find long run supply for costs $C_x(q) = x + q^2$. (Hint: Elegant answer)



Long Run vs. Short Run Supply with Heterogeneous Firms





- Continuous firms allows us to compute supply by integration!
- Long run supply is all supply (😬) by inframarginal firms $x \leq x(P)$:

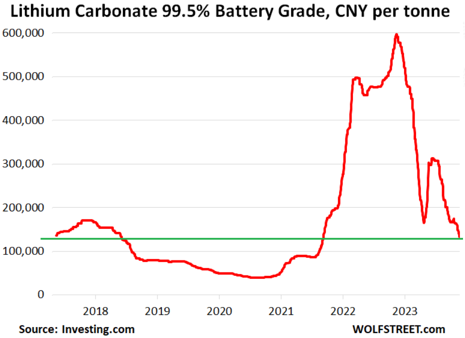
$$Q_S^{LR}(P) = \int_1^{x(P)} q_x(P)dx = \int_1^{P/2} P/(2x^2)dx = [P/2][-x^{-1}]\Big|_1^{P/2} = \frac{1}{2}P - 1$$

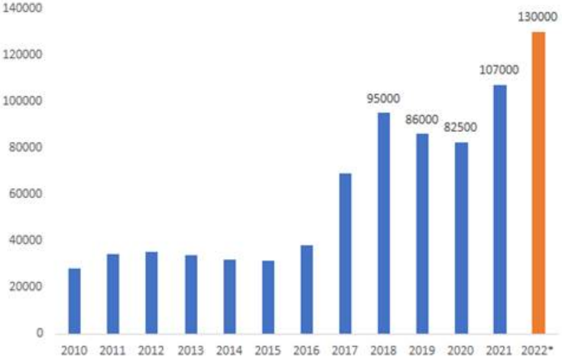
- This integral [or "mass" or "measure"] is well-defined for $P \geq 2$.
- The supply curve now rises due to cost convexity and heterogeneity
- Market supply is more elastic than firm supply
- Short run supply starting at a price P_0, i.e. with marginal seller $x(P_0)$:

$$Q_S^{SR}(P|P_0) = \int_1^{x(P_0)} P/(2x^2)dx = [P/2][-x^{-1}]\Big|_1^{x(P_0)} = (P/2)[1-2/P_0]$$

Thinker Q: Natural Resources Tend to be Price Volatile

- Their supply tends to be inelastic, since a well or mine has been dug, and extraction costs are lower
- What supply and demand shifts led to this price rise?





Concluding Thoughts on Extensive and Intensive Margins

- We just fleshed out the logic for supply curves
- Demand with Heterogeneous Consumers:
  - If supply increases and so price falls, the new consumers like the good less and prior consumers buy more
  - Demand elasticity is higher accounting for entry
  - Smart phones: inframarginal consumers buy the fancier phones