Hidden Passing Games[*]

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Abstract

We introduce random encounter matching games, where a hidden trait is sometimes unwittingly passed — either private and bad (like counterfeit money), or collective and bad (a disease), or hidden and good (rare coin), or collective and good (information). One optimally expends effort to avoid acquiring bad traits or trying to acquire good ones. The game shifts from strategic complements to substitutes changing from private to collective, or good to bad traits.

A unique equilibrium exists in all cases, and coincides for collective bad and private good traits, and collective good and private bad traits. Equilibrium incidence can be a misleading signal of prevalence for bad traits at high prevalence: One cannot infer that counterfeiting is less severe when there is less passed counterfeit money, or prevalence is higher with more infections.

*This is a polished version of a manuscript Contagious Matching Games prepared for a presentation by Lones at the "Search and Matching Workshop" on December 16, 2006 at University of Pennsylvania. The Editors suggested this more appropriate title, and many other improvements. Its model and results are the same. We thank Philipp Kircher for inviting us, and for his feedback in the talk. It has since been polished for motivation, but no results have been added.

†*Elena sadly died in summer 2021*, while on tenure track in the Robert C. Vackar College of Business & Entrepreneurship at University of Texas Rio Grande Valley. :(Lones dedicates this paper to her, which reflected her insight that our counterfeiting project (published in *Econometrica* in 2015) had implications for contagions.

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1 Introduction

What do counterfeit money and AIDS have in common? The answer is that both "traits" change hands when people meet, and are unwittingly acquired through lack of care. By contrast, hidden desired traits like rare valuable stamps are caught more often by paying more attention. We introduce and explore a tractable new class of pairwise games with random pairwise meetings and costly vigilance impacting the chance that a positive or negative attribute trait is secretly passed along. It may be a private bad, like counterfeit money, stolen goods, forged art — or mortgage backed securities with overstated safety in the 2008 financial crisis. Or, it may be a collective bad, like a disease or computer virus. On the other hand, it night be a good trait, and private, like a 1943 copper cent, or collective, like prized information scoops.

In this new class of 2×2 random matching games, people optimally expend costly effort to acquire a good trait, or avoid a bad trait, that invisibly passes in meetings. We assume additive utility over vigilance costs and the trait's expected harm or gain. With collective bad traits, vigilance efforts are strategic substitutes — people try harder when they expect others try less. Conversely, with private bad traits, vigilance efforts are strategic complements — vigilance rises in others' vigilance. The opposite taxonomy emerges for private and collective good traits.

Optimal behavior reflects prevalence while incidence reflects both behavior and prevalence. By assuming geometric vigilance costs, we can fully solve all four games, finding a unique Nash equilibrium in each case. We compute trait dynamics and steady state stocks. We ask if higher passing flows correspond to higher hidden stocks. For instance, does more passed counterfeit money signal a higher counterfeit rate? Or is disease incidence higher when prevalence is higher? The intuitive link is unclear because people strive harder not to acquire a disease or fake money when it's more prevalent. At low stocks, stocks and flows comove. But at higher levels, a flow moves opposite to the stock. For instance, at greatly inflated counterfeiting rates, larger passed money rates signify less counterfeit money. For a lower counterfeit chance reduces vigilance so much that more counterfeits pass by the first person screening; this can swamp the fall in the screening chance by the next. In the games with a private or collective bad trait, this substitution effect can reverse the incidence-prevalence link. With a private or collective good trait, behavior is identical, respectively, to a collective or private bad trait, but the incidence is increasing in prevalence.

This paper introduces the class of hidden passing games with costly vigilance and equilibrium passing rates. It arose from thinking of our counterfeiting paper Quercioli and Smith (2015) as a private bads case in this 2×2 matching games taxonomy with a costly vigilance choice.[1] We know of one prior paper that relates to our collective bad contagion game: Kremer (1996) introduces choice of number of partners into an AIDS contagion model. Rather than restrict to partner choice, we are agnostic about contagion details, and posit costly avoidance actions limiting infection risk. Seeing our presentation, Greenwood et al. (2019) explored the AIDS contagion in Africa, but with forward-looking individuals (updating their AIDS status); their ambitious calibrated analysis is focused on policy analyses. This manuscript inspired Keppo et al. (2023), which instead finds that a constant elasticity cost function yields a tractable log-linear modification of the SIR dynamics that incorporate equilibrium avoidance.

To highlight the similarity across cases, we eschew generality, and assume geometric costs and homogenous agents. Our strategic analysis of these matching games will exploit how vigilance impacts payoffs and the evolution of the latent unobserved hidden trait, which feeds back to impact optimal vigilance. We shall focus on bad traits, and then deduce good traits by corollary in §5. All proofs are so simple they are in text.

2 The Model

People in a unit continuum $[0, 1]$ randomly meet pairwise in continuous time at a flow rate one. Consider some *trait*, either bad like counterfeit money or a disease of cost v if acquired, or valuable, a rare stamp or information scoop worth v. Assume it is currently held by an assumed known fraction $\gamma \in (0, 1)$ of the population — the *prevalence*. If not known with certainty, assume γ is its expectation. The trait might change hands unnoticed in an encounter. Everyone exerts costly *(vigilance) effort* $e \in [0, 1]$, which will be the trait discovery probability, conditional on a meeting.

We give a unified treatment of four economic settings in a 2×2 taxonomy with a representative agent: The trait may be bad or good, and private or collective (i.e. retained when passed on). Counterfeit money is a private bad: undesired, and lost when passed on. A disease is a collective bad: No one desires it, and retained when passed on. A rare coin is a private good: desirable, and lost if passed on. Finally, information scoops are collective good traits: desirable, and retained if passed on.

For the private bad traits examples, our game payoff below merges consecutive

[1] For our laser focus on four games, we ignore costly counterfeit quality, and general cost functions.

transactions involving the same possibly fake note with face value $v > 0$. Since passing a note leads to immediate seizure if discovered, any seller has an incentive to examine notes she expects might be examined. Assume seller i expends effort e_i scrutinizing each note she is handed. Then the buyer takes a loss v if he misses that it is fake, and then the next buyer j discovers it — with independent discovery events. If a fraction $\gamma \in (0,1)$ of notes are counterfeit (namely, the *counterfeiting rate*), then seller i's expected loss is

$$\gamma(1 - e_i)e_j v \tag{1}$$

Effort $e = 1$ chokes off all passing here and in all cases below.

Ponder the case of a hidden private good of bonus value $v > 0$ — for instance, a rare coin purchased. Our game merges consecutive transactions for the same possibly valuable good. To simplify analysis, and preserve a parallel thought experiment for private bad traits, we assume traders only examine goods before selling, so as not to lose any hidden gems.[2] Vigilance effort e_i reduces the chance of trading it away. Assume a fraction $\gamma \in (0,1)$ of all stamps or coins are highly valuable. If trader j misses it and trades it to i who finds it when he sells, then trader i wins a prize v. This has chance $(1 - e_j)e_i$, by independence of the discovery events. Then buyer i's expected gain is

$$\gamma(1 - e_j)e_i v \tag{2}$$

Next, consider a hidden collective bad with loss $v > 0$, like a contagious disease. Person i may catch the disease from another infected person j when j sneezes, and neither individual covers his mouth. We assume interactions in ignorance of one's infection status, with no one purposely infecting others.[3] In this way, roles in encounters are symmetric, and one expends the same effort in any encounter. Assume a prevalence $\gamma \in (0,1)$. Then one catches the disease in the triple event that one is infected and neither player's vigilance inhibits its passing — namely, chance $(1 - e_i)(1 - e_j)$ in an encounter of i and j, by independence. Individual i's expected loss is[4]

$$\gamma(1 - e_i)(1 - e_j)v \tag{3}$$

[2]Realistically, buying incentives are not to acquire lemons, rather than seeing if one is perchance buying a gem. Think of the mortgage backed security trading game that led to the Financial Crisis.

[3]This simplifying assumption is realistic for many diseases when latent and only revealed by tests until symptoms emerge (and for some diseases, knowingly passing it has been deemed a criminal act).

[4]If infection confers immunity (SIR), the simple model must assume the infected status is known, and we ignore meetings with the immune. If past infection does not confer permanent immunity (as with COVID), then this parable assumes the extreme SIS case where re-infections can occur.

	collective consumption	private consumption
"bad"	$-\gamma(1-e_i)(1-e_j)v - C(e_i)$ (submodular game)	$-\gamma(1-e_i)e_j v - C(e_i)$ (supermodular game)
"good"	$\gamma e_i e_j v - C(e_i)$ (supermodular game)	$\gamma e_i(1-e_j)v - C(e_i)$ (submodular game)

Table 1: **Four Quasilinear Vigilance Matching Games.** Payoffs of individual i choosing effort e_i in a meeting with another individual j choosing effort e_j, where the stakes of a mistake are v. For private traits, i is a seller for a bad trait, and a buyer for a good trait. For In the collective trait case, i and j have symmetric roles.

Finally, consider a collective good, like a stock tip, worth v. As with collective bad traits, the game is symmetric, with people choosing how hard and revealingly to interact — namely, effort e. An effusive person will pass on and absorb information more often. In this case, if the prevalence of valuable stock tips is γ, then each acquires the information scoop from the other with chance $\gamma e_i e_j$. Individual i's expected gain is, by independence:

$$\gamma e_i e_j v \tag{4}$$

Toggling between good and bad, or private and collective, the matching game switches between strategic complements and substitutes — namely, supermodular and submodular games in Table 1. So optimal vigilance increases in the partner's effort for collective good and private bad traits, and falls for collective bad and private good traits. This strategic distinction has economic importance. For in supermodular games, the comparative statics impact of a rise in fundamentals v are amplified relative to an individual optimization. For if v rises, then everyone's effort rises, and then rises more given others' higher efforts (ad infinitum, until it equilibrates). This describes collective good and private bad traits. Likewise, in submodular games, the comparative statics impact of a rise in fundamentals v is attenuated relative to an individual optimization. For if v rises, then each individual responds with more effort, and then responds negatively to the other's increased effort (ad infinitum, until it equilibrates). This is the story of collective bad and private good traits.

Assume effort $e \in [0,1]$ costs $C(e)$, a smoothly increasing convex cost function, with $C(0) = 0$. In this paper, we seek sharp results on flows and stocks. To this end, we posit $C(e) = ce^r/r$, where $r > 2$ and $c > 0$ is the marginal cost of effort at $e = 1$. We assume geometric costs for simplicity in our parable, as it allows easy closed form derivation focused on the economics of this new class of quasilinear games.

3 Private Bad Traits

3.1 Equilibrium

Everyone acts myopically, as they only cannot impact the future prevalence or other players' behavior given the continuum assumption, or impact their future payoffs, recalling footnote (4). Thus, everyone myopically best responds, and Nash equilibrium at each moment in time is the appropriate equilibrium concept. In the collective bad traits case, the simplicity assumes people do not are equally harmed by subsequent infections. The simplicity of the analysis owes to using static Nash equilibrium at each time, and ignoring dynamics. We also rule out asymmetric and mixed equilibria.

First consider private bad traits, like counterfeit money. At a meeting, each individual i minimizes the losses (1) plus expected infection costs $C(e_i)$. Crucially, players are only impacted by the average population efforts $\bar{e}$ of others he meets. So let's define the payoff function[5]

$$\pi(e, \bar{e}) = -\gamma\bar{e}(1-e)v - ce^r/r \tag{5}$$

The equation $\pi_{\bar{e}}(e, \bar{e}) = 0$ defines the best response individual effort e to average effort $\bar{e}$. Since $\pi_{\bar{e}e}(e, \bar{e}) = \gamma v > 0$, higher efforts by others raises the marginal product of own efforts, so that game is supermodular (see Table 1). For instance, the more carefully others examine the currency you pass to them, the more scrutiny one wishes to apply the money one is handed. Further, this supermodularity is greater at higher prevalence γ. As in Diamond (1982) this can lead to multiple equilibria.

Using the first order condition, $\gamma\bar{e}v = C'(e) = ce^{r-1}$ is solved by the same effort e for all players, and so is the mean $e = \bar{e}$. So there are no mixed strategy equilibria. The second order condition holds in our paper, when $r > 1$, for then costs are strictly convex and benefits are linear in effort e. The *equilibrium first order condition* becomes

$$\gamma\bar{e}(\gamma)v = c\bar{e}(\gamma)^{r-1} \Leftrightarrow \gamma v = c\bar{e}(\gamma)^{r-2} \Leftrightarrow \bar{e}(\gamma) = (\gamma v/c)^{\frac{1}{r-2}} \tag{6}$$

A final possibility is a corner solution at $e = 0$ or $e = 1$. Such does not exist when $r > 2$ and $\gamma v \leq c$, for then $\gamma\bar{e}v = ce^{r-1}$ is solvable with $\bar{e}^{r-2} = \gamma/c \leq 1$. On the other hand, with not too convex costs, $1 < r < 2$, there is a corner solution $e = 1$ for small

[5] This is a flow payoff. To be precise, imagine a time interval $[t, t+dt)$ where efforts $(e, \bar{e})$ are fixed. Then the payoffs in that interval are $\pi(e, \bar{e})dt$. The mathematics only makes sense if efforts lie in $[0, 1]$, but we only $\gamma > 0$ because the assumed unit meeting flow cannot be identified in this model.

$\gamma > 0$. Assuming strictly convex marginal costs with $r > 2$ precludes this possibility, and allows us to simply consider the interior solution: $\bar{e}(\gamma) < 1$ when $\gamma v < c$.

Altogether, there is a unique optimum of $\pi(e, \bar{e})$ in e, and thus everyone chooses effort $e = \bar{e}$. This means our representative agent model yields a unique prediction:

Proposition 1 *Assume a private bad and strictly convex marginal costs ($r > 2$). There exists a unique Nash equilibrium, and it is symmetric and in pure strategies. The common vigilance effort $\bar{e}(\gamma)$ rises in the counterfeiting rate γ.*

Proof: Of course, we have an explicit formula for vigilance effort, but we can log-differentiate the equilibrium FOC (6) to deduce that effort rises in prevalence:

$$\bar{e}'(\gamma) = \frac{\bar{e}(\gamma)}{(r-2)\gamma} > 0 \tag{7}$$

Intuitively, people individually wish to increase effort with greater prevalence, and is amplified — due to the strategic complements — by the greater effort by others.

We next link the flow of passed money rates to the stock of counterfeit money. We consider in §3.2 and §3.3 two scenarios — with fixed and endogenous prevalence.

3.2 The Market Response to Infections

For our prevalence analogue, assume a fixed counterfeiting rate $\gamma \in (0, 1)$. Dividing (1) by the note value v, the equilibrium flow of passed money (newly discovered counterfeit notes or "incidence) is:

$$I(\gamma) = \gamma \bar{e}(\gamma)[1 - \bar{e}(\gamma)] \tag{8}$$

This is the chance γ that a note is counterfeit, *and* this is missed by the last holder, *and* then it is noticed (the last product). This product vanishes with zero vigilance effort, for then no one ever finds the bad. Equally well, it vanishes with perfect screening, as the fake note is never passed on. When prevalence γ rises to c/v in (6), the equilibrium calls for effort $\bar{e} = 1$, choking off new passed money. So the peak counterfeiting rate corresponds to a minimum passed money. We now explore the generality of this insight.

Passed money requires inattention by one buyer, and diligence by the next ($0 < \bar{e}(\gamma) < 1$). Now, $\bar{e}(\gamma)[1 - \bar{e}(\gamma)]$ rises in $\bar{e}(\gamma) < 1/2$, and by Proposition 1, the vigilance effort obeys $\bar{e}'(\gamma) > 0$, if $r > 2$. Easily, passed money (8) rises too, or $I'(\gamma) > 0$. We focus on the surprising case when the passed rate falls as the counterfeiting rate γ rises, due to the rising vigilance effort, amplified by the supermodular game effect.

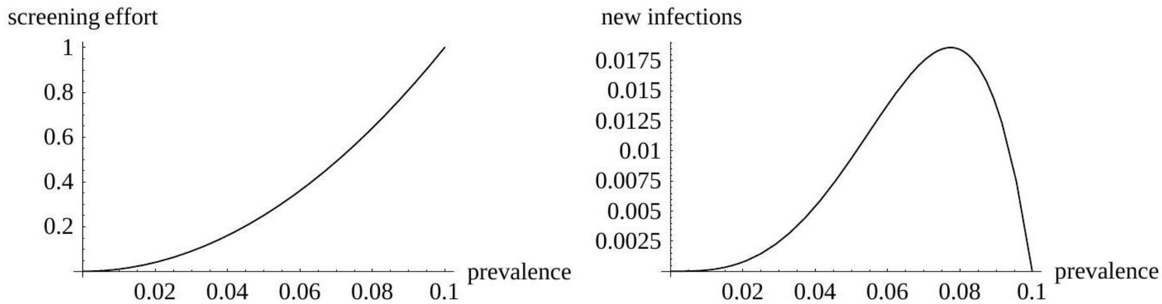


Figure 1: **Private Bad Traits.** Vigilance effort rises in the stock γ (Proposition 1). The plot assumes cost elasticity $r = 2.5$ and parameter $c = 0.1$. With a constant screening, passed money (8) would rise in γ. But with effort rising in response to greater γ, the peak passed rate is at $\gamma \approx 0.08$, as Proposition 2 asserts.

Proposition 2 *Assume a private bad, and strictly convex marginal costs $r > 2$. Passed money falls when the counterfeiting rate γ rises if $\gamma \in (0, c]$ is high enough.*

Proof: Differentiate incidence (8), substituting (6) into (7), to get:

$$I'(\gamma) = \bar{e}(\gamma)(1 - \bar{e}(\gamma)) + \gamma(1 - 2\bar{e}(\gamma))\bar{e}'(\gamma) = \bar{e}(\gamma)(1 - \bar{e}(\gamma)) + \frac{(1 - 2\bar{e}(\gamma))\bar{e}(\gamma)}{r - 2} \quad (9)$$

Simplifying, passed money falls in the counterfeiting rate, or $I'(\gamma) < 0$, iff $\bar{e}(\gamma) > 1 - 1/r$. Given $\bar{e}(\gamma) = (\gamma v/c)^{\frac{1}{r-2}}$, this holds iff $\gamma v > c(1 - 1/r)^{r-2}$. The right side rises to c as $r \downarrow 2$, and so for every $r > 2$, there is $\varepsilon > 0$ with $I'(\gamma) < 0$ for $\gamma v > (1 - \varepsilon)c$. □

Now let's consider implications of this result for policy changes. The government usually wishes to reduce counterfeiting by encouraging vigilance effort. For instance, a more unique currency feel lessens the cost parameter c. This raises the vigilance effort $\bar{e}(\gamma)$ in (6) only if $\bar{e} < 1/2$. So Proposition 2 says that infections move in opposition to the counterfeiting rate γ when the vigilance effort exceeds $1 - 1/r > 1/2$ — which eventually holds for large γ, by the logic of the proof of Proposition 2. In this case, policies to encourage vigilance effort are counterproductive.

Large counterfeiting rates that lead to this possibility are not without precedent. For instance, during the American Revolution, the British so successfully counterfeited American money that the Continental currency soon became worthless — hence the saying "Not worth a Continental." The Secret Service reports that later on, during the Civil War, one-third to one-half of the circulating currency was counterfeit. Some elite goods have a large share of counterfeit goods passed as the real deal.

3.3 The Prevalence Dynamics

We now allow more realistically that the counterfeiting rate γ is an equilibrium object. For one might wonder if it is meaningful to premise Proposition 2 on its changes. Assume a fixed inflow $\delta > 0$ of new counterfeits in continuous time $[0, \infty)$. For instance, this may be new counterfeits or forged art entering the economy. Since infections are exits from the prevalence pool, dynamics are by (8):

$$\dot{\gamma} = \delta - I(\gamma) = \delta - \gamma\bar{e}(\gamma)[1 - \bar{e}(\gamma)] \tag{10}$$

Since $\bar{e} \equiv (\gamma v/c)^{\frac{1}{r-2}}$ by (6), the steady-state level of the bad trait obeys $\dot{\gamma} = 0$, and thus

$$\bar{e}^{r-1}(1 - \bar{e}) = \delta v/c \tag{11}$$

The peak is $\bar{e}^* \in (\bar{e}_1, \bar{e}_2)$. Then $\dot{\bar{e}} > 0$ on $(0, \bar{e}_1)$ and $\dot{\bar{e}} < 0$ on $(\bar{e}_1, \bar{e}_2)$ and $\dot{\bar{e}} > 0$ on $(\bar{e}_2, \infty)$. Thus, $\bar{e}_1$ is the stable root. Ignore $e > \bar{e}_2$, where counterfeiting rate spirals out of control. If $e < \bar{e}^*$, or the counterfeiting rate γ is below $c\bar{e}^*$, then incidence $I(\gamma)$ and the counterfeiting rate γ both rise or both fall. But suppose a large cache of new fake money enters circulation and γ jumps to $(\bar{e}^*, \bar{e}_2)$. Then initially incidence $I(\gamma)$ rises as the counterfeiting rate γ falls. Thus, Proposition 2's finding — more passed counterfeits observed and yet a lower counterfeiting rate can occur — is a robust insight for equilibrium models with a fixed inflow of new counterfeits, and an endogenous counterfeiting rate.

4 Collective Bad Traits

4.1 Equilibrium

We now consider collective bad traits, like a contagion, with payoffs (3). At each moment, faced with prevalence γ and others' mean effort $\bar{e}(\gamma)$, someone who is susceptible minimizes expected infection costs and vigilance costs:

$$\gamma(1 - \bar{e}(\gamma))(1 - e)v + ce^r/r$$

with respect to vigilance effort e. This yields the equilibrium FOC of the contagion game:

$$\gamma(1 - \bar{e}(\gamma))v = c\bar{e}(\gamma)^{r-1} \tag{12}$$

Hence, $\bar{e}(0) = 0$, given $r > 1$. As in the private bad traits case in §3.1, the second order condition holds given strict cost convexity $r > 1$, since gains are linear in effort.

Proposition 3 *Assume a collective bad and strictly convex costs ($r > 1$). There exists a unique Nash equilibrium, and it is symmetric and in pure strategies. The vigilance effort $\bar{e}(\gamma)$ rises in prevalence γ.*

Proof: We can no longer explicitly solve the equilibrium FOC, as we did in §3.1. Instead, differentiating (12) in γ yields $(1 - \bar{e}(\gamma))v - \bar{e}'(\gamma)\gamma v = c(r-1)\bar{e}(\gamma)^{r-2}\bar{e}'(\gamma)$. Simplify this derivative, by substituting back from (12):

$$\bar{e}'(\gamma) = \frac{(1 - \bar{e}(\gamma))v}{\gamma v + c(r-1)\bar{e}(\gamma)^{r-2}} = \frac{\bar{e}(\gamma)(1 - \bar{e}(\gamma))}{\gamma[\bar{e}(\gamma) + (r-1)(1 - \bar{e}(\gamma))]} > 0 \tag{13}$$

Notably, the convexity threshold ($r > 1$) for an increasing equilibrium vigilance effort is less with collective bad traots here than with private bad traits ($r > 2$) in Proposition 1. To understand why vigilance more readily rises in prevalence in a collective good traits setting, note that strategic forces for monotonicity now reinforce individual incentives. For with collective bad traits, actions are strategic substitutes: The more carefully one protects against disease transmission, the more one lowers the marginal product of others' efforts. In other words, greater vigilance effort lowers the marginal product of others' actions — it is a submodular game (see Table 1). If perchance others' efforts fell as prevalence rose, then this would amplify the individual incentive for greater effort.

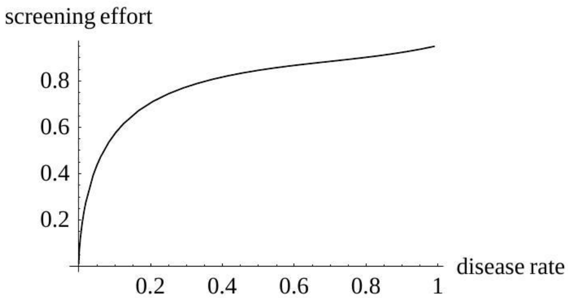
4.2 The Market Response to Infections and Disease Dynamics

Fix the prevalence γ, namely, the chance that someone is contagious. New infections happen when a contagious and a susceptible person meet (chance $\gamma(1-\gamma)$), and the disease passes. If we assume that efforts do not reflect infection status (since symptoms have not yet been obvious), then by (3), the equilibrium flow of new infections is:

$$I(\gamma) = \gamma(1-\gamma)(1 - \bar{e}(\gamma))^2 \tag{14}$$

The key strategic difference with (8) is the effort interaction.

Proposition 4 *Assume a collective bad, and strictly convex costs $r > 1$. Incidence rises iff prevalence falls, for all $\gamma > 1/2$.*



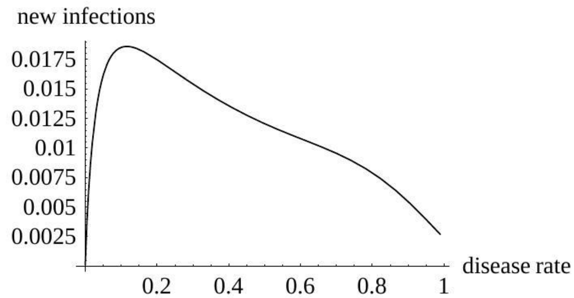


Figure 2: **Collective Bad Traits.** Vigilance effort rises in prevalence γ (Lemma 3). The parameters are as in Figure 1. With a constant vigilance effort, new infections will peak at $\gamma = 0.5$, when $\gamma(1-\gamma)$ is maximized. But in this strategic model, effort rises in prevalence γ, and accordingly incidence peaks at a lower γ, here just over 0.1. This is well below the threshold guaranteed by Proposition 4.

Proof: Substituting from (13), the derivative of new infections (14) equals

$$\begin{aligned} I'(\gamma) &= (1-2\gamma)(1-\bar{e}(\gamma))^2 - 2\gamma(1-\gamma)(1-\bar{e}(\gamma))\bar{e}'(\gamma) \\ &= (1-\bar{e}(\gamma))^2 \left[1 - 2\gamma - \frac{2(1-\gamma)\bar{e}(\gamma)}{\bar{e}(\gamma) + (r-1)(1-\bar{e}(\gamma))}\right] \end{aligned}$$

As the subtracted last term is positive if $r > 1$, we have $I'(\gamma) < 0$ when $\gamma > 1/2$. $\square$

Assume finally an exit rate (deaths or recovery) $\delta > 0$ of those infected with the contagious trait. Modifying the standard disease dynamics using (14), we find that

$$\dot{\gamma} = I(\gamma) - \delta\gamma = [1-\bar{e}(\gamma)]^2\gamma(1-\gamma) - \delta\gamma = \left[1 - (\gamma/c)^{\frac{1}{r-2}}\right]^2 \gamma(1-\gamma) - \delta\gamma \qquad (15)$$

Assume $0 < \delta < 1$ and $r > 2$. Then the left side of (15) is strictly falling from 1 to 0 on $(0, \min(1, c))$, and so (15) has a unique root $\bar{\gamma}$. So if $\gamma > \bar{\gamma}$, then $\dot{\gamma} < 0$ — namely, prevalence falls. By Proposition 4, incidence rises for $\gamma > 1/2$. Thus, Proposition 4 is a robust insight for equilibrium models when the prevalence is endogenous.

5 Private and Collective Good Traits

First consider private good traits. Individual i maximizes his expected gains (2) in effort e_i, given others' expected effort $\bar{e}$, less his vigilance costs. Thus, his effort e maximizes

$$\gamma(1-\bar{e})ev - ce^r/r$$

Notably, *the first order condition* $\gamma(1-\bar{e})v = ce^{r-1}$*, and thus optimal behavior, is identical to the case of collective bad traits.* Table 1 explains why, since individual i's payoffs in these two submodular games only differ by an additive term independent of his own effort e_i. In other words, the unrelated economic games — collective bad traits and private good traits — share the equilibrium effort solving (12), and so have the same predictions Proposition 3: Namely, effort is always rising in prevalence.

Next, the discovery flow of the diamonds in the rough reflects their prevalence γ, and the equilibrium passage chance. By (2), the equilibrium incidence is now:

$$I(\gamma) = \gamma\bar{e}(\gamma)(1-\bar{e}(\gamma)) \tag{16}$$

Incidence always rise in prevalence γ, unlike in Proposition 4 with collective bad traits. Indeed, substituting from (13), new infections (5) have slope:

$$I'(\gamma) \quad = \quad \frac{\bar{e}(\gamma)(1-\bar{e}(\gamma))^2 r}{\bar{e}(\gamma)+(r-1)(1-\bar{e}(\gamma))} > 0$$

Just as in §3.3, the prevalence γ of good traits can be an equilibrium object. Given an inflow $\delta > 0$ of new diamonds in the rough in $[0,\infty)$, we solve $I(\gamma) = \delta$, as in (6).

Finally, consider collective good traits, like information scoops. In the matching game, individual i maximizes his payoffs (4) in effort e, fixing others' expected efforts $\bar{e}$:

$$\gamma e\bar{e}v - ce^r/r$$

Here too, the first order condition, $\gamma\bar{e}v = ce^{r-1}$, implies the identical equilibrium effort as (6), for the diagonally opposite private bad traits case. Namely, $\bar{e}(\gamma) = (\gamma v/c)^{\frac{1}{r-2}}$. This makes sense from Table 1, since the payoffs of individual i in these supermodular game only differ from an additive independent of his effort e_i.

Finally, inspired by (2), the equilibrium incidence $I(\gamma) = \gamma\bar{e}(\gamma)\bar{e}(\gamma)$ has slope

$$I'(\gamma) = \frac{r\bar{e}(\gamma)^2}{r-2} > 0$$

after substituting from the formula for $\bar{e}'(\gamma)$ in (7). So unlike Proposition 2, the incidence of information scoops is increasing in prevalence. So while the equilibrium efforts are the same, the prevalence dynamics are not.

6 Conclusion

Inspired by the matching paper of Mortensen (1982), we have written this as a simple parable. We introduce a class of passing games of hidden traits in random matching models, in which incentives reflect estimated prevalence rates of a trait — like a counterfeit note, a contagious disease, a prized stamp, or a valuable information nugget. These can be private or collective traits, as well as good or bad. Ours is also a new 2×2 class of games that toggles between supermodular and submodular, as well as between positive and negative externalities.

In all cases, static Nash equilibrium correctly captures fully rational dynamic play, given a continuum of players. It is not unreasonable to infer unobserved stocks from flows; with constant behavior, this would be justified. But equilibrium efforts reflect stocks, and skew flows. For private and collective bad traits, incidence flows may fall when stocks rise, or conversely. This respectively reflects the supermodularity and submodularity. But for private and collective good traits, higher incidence does correctly signal higher prevalence.

A referee has suggested that community vigilance norms are an intriguing research avenue, since they could help others in public trait passing games. Notably, however, if done en masse, would impact others in the private trait passing games, by impacting prevalences: So all passing games are public. If only we all checked our money more carefully, the counterfeiting rate would fall. This no doubt underlies efforts by the Secret Service to ensure we check our money.

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