The Economics of Web Search

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Abstract

We introduce a tractable model of sequential search or choice among distinct pre-ordered options for web search or informed search. Payoffs are the sum of a random *known factor* and a random *hidden factor*, learned after inspection. In this nonstationary search model, prior options are sometimes recalled. To capture search engines, we assume Gaussian factors, where noise shifts from the hidden to the known factor as web search accuracy rises. We prove:

1. Search lasts longer with more payoff *dispersion*, and thus with more disperse hidden factors rises. But dispersion of known factors reduces search duration.
2. The search stopping chance rises over time with log-concave factor densities
3. The chance of recalling options rises, and older ones are recalled more often
4. Search lasts longer with more options, since searchers grow more ambitious
5. The marginal value of web search accuracy is higher eventually than initially
6. With a thin factor density tail, the limit recall chance is boundedly positive

Item 1 solves a long open search theory puzzle. Items 2–4 are new results for ordered search. Item 5 uncovers a possible natural monopoly in web search. By Item 6, stationary search might not approximate search with many options.

Keywords: sequential / nonstationary search, duration, logconcavity, dispersion

JEL codes: D81, D83

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1 Introduction

Most search now occurs in mediated web search environments, through smartly sorted lists online, like Netflix movie quests, Google or Amazon searches, or even using radio station presets. With generative AI, assisted search will surely grow in importance. Ordered search fundamentally differs from stationary search — for quitting and recalling both play key roles. In the 100,000 Google searches per second, or three trillion per year, rarely does one choose the most recently-searched web page, and many searches are abandoned. Neither stopping, quitting, nor recall occur in stationary search models. Web search aside, search rarely transpires in the zero information vacuum of random stationary search. Assuming ex ante identical options is almost always unjustified. For search is invariably at least partially informed — people know which stores sell higher quality; firms hiring workers can easily observe their college of origin; and those seeking romantic partners quickly perceive looks or location.

To analyze this richer world, we introduce and solve a simple new search model with finitely many ex ante heterogeneous risky options. In our twist, each payoff is the sum of a random *known* and *hidden* factor. A search engine or prior knowledge sorts by known factors. A searcher, whom we call Sam, sees all known factors, but only learns the hidden factor after paying a look-see cost. Sam eventually takes one option or quits searching. He optimally explores options in order of known factors (maybe inferred from Google), proceeding until either quitting, recalling a prior option, or exercising the current one. For any given set of known factors, our model is Weitzman's 1979 "Pandora's Box" dynamic programming problem. Think of Google's Page Rank algorithm as finding known factors, creating a random Weitzman problem. Weitzman found no comparative statics except for riskier options, but we derive many predictions for our random two factor version model when both factors have log-concave distributions — an assumption met by common continuous distributions.

In stationary wage search, the value function is first horizontal, and then the 45^o diagonal. So its slope is the acceptance chance — zero then one. In our nonstationary world, the value function is increasing and convex in the fallback prize, and the slope has richer economic meaning: the chance of eventually taking the option (Figure 2).

Our paper fully characterizes search behavior over time, and as model parameters vary. First, pre-search information and optimal recall go hand–in-hand, and we think ours is the first general theory characterizing recall rates.[1] Our model is designed for estimation, predicting many aspects of Sam's evolving behavior over time. He is initially forward-looking, torn just between accepting the current option and passing. The best fallback option weakly improves over time, while future prospects dim as the best options are exhausted. At some point, passing is dominated, and search ends. Log-concavity ensures that quitting, recalling, and choosing the current option intensify over time: Sam grows more willing to stop and choose the current option, or recall an earlier one, or quit searching (Theorem 1). Further, when Sam does recall, he reaches back to the earlier options more often than later ones (Theorem 2).

Second, we turn to an open and fundamental question about standard stationary search: What stochastic changes raise search duration (Mortensen, 1987)? This is subtle because Sam optimally searches more aggressively with a richer set of options. So if the prizes stochastically improve, the reservation threshold rises, but is offset by more weight in all upper tails. So search duration rises only if the "substitution effect" of a higher reservation prize dominates the direct effect of better options. Does it?

We show that the *dispersion* stochastic order — namely, if the gap between any two percentiles increases everywhere — generally resolves this tradeoff.[2] Namely, duration is higher with a more dispersed prize distribution for the undiscounted stationary search model (Theorem 3).[3] Intuitively, the substitution effect of more aggressive search swamps the direct effect of more high prizes. The dispersion order notably differs from a mean-preserving spread that the search literature has long focused on.[4] This result is so significant we offer two arguments, one with calculus and one without.

Now consider how the dispersion insight plays out in our two factor search model. First, search duration rises when the hidden factor grows more disperse (Theorem 4) — the substitution effect of more ambitious search swamps the direct effect of better prizes. The logic is more subtle in this nonstationary setting, where the rank-ordered

[1]We name our searcher Sam for Karlin (1962), who solved the first sequential search problem: It was nonstationary due to a finite horizon. His options were identical, and disallowed recall. Sam is also an acronym for "search and matching", the main topical playground for these methods.

[2]The dispersion order usefully dovetails with log-concavity: For truncating a log-concave distributions makes it less disperse, and the sum of log-concave r.v.s is more dispersed than either.

[3]A trickier analysis is in McCall's (1970) discounted wage search model (see here for details).

[4]Dispersion is empirically measurable, and typical distributions are indexed by it (see Table 1).

options deteriorate faster with greater dispersion of the known factor. Sam is less willing to continue search with a more disperse known factor (Theorem 5).

Third, we offer a novel theory of variable accuracy web search engines. For this, we restrict to Gaussian factor payoffs, so that rising *accuracy* transfers variance from the hidden to the known factor. The search substitution effect is strong: as accuracy rises from zero, Sam relaxes his search intensity so quickly that his search outcome actually worsens (Theorem 6): *A fortiori*, a poor search engine is worse than none at all. We also discover that for larger quit options, as accuracy rises, the chance that Sam chooses to search falls, and the probability he quits searching rises (Theorem 7).

A more accurate search engine is more informative in Blackwell's sense, an extension of the nonconcavity of the value of information[5] now plays an economically important role. Theorem 8 asserts that the marginal value of accuracy is increasing for low search costs, but this result extends here all the way to perfect accuracy. The rising marginal value of information means that search engines may be a natural monopoly — a now topical issue. Also, the marginal gains to accuracy or lower search costs are larger with more options. The vast growth of the web has propelled the social gains to better search engines (Corollary 2), strengthening the natural monopoly.

Fourth, we ask how search changes in the number of options. Web matching markets, like match.com, boast of large mating pools. While higher quality mates are presented first, Sam searches even longer with more options, by Theorem 9. For Sam's search standards rise faster than do the options stochastically improve. Once more, the substitution effect swamps the direct effect: For with log-concavity, gaps between order statistics of known factors shrink with more options, spurring search.

We finally let the number of options explode. The benchmark stationary search model (McCall, 1970) is justified in a finite world only if it well-approximates search with many options — an arbitrarily long Google search list here. We question this premise. For while the conditional chance of recalling a prior option falls in the number of options, it only vanishes in the limit if the known factor distribution has a "thin tail" (Theorem 10). Otherwise, the recall chance never vanishes since top order statistics of known factors have boundedly positive gaps. So the stationary search model fails to approximate search without a thin tail, as the chance Sam recalls a

[5]Found by Radner and Stiglitz (1984), a clear theoretical model is in Chade and Schlee (2002).

prior option does not vanish with many options. For example, top order statistics of the exponential distribution have constant gaps, irrespective of the number of options.

Since Google search lists, consumer preferences, and information are typically unobservable to econometricians, our theory must overcome selection effects: Arriving at a later stage offers more damning evidence of Sam's already-explored options: And if prior known factors are weaker, so too is the next one, and thus it is less likely that search stops. The selection intuitions run counter to the conclusion of Theorems 1 and 2 that Sam's stopping and quitting grows over time. We show that log-concavity prevents selection from swamping the direct effect of falling known factors.

LITERATURE. We add to the pure theory of sequential search. The first model of nonstationary web search and recall with different options is Weitzman's 1979 "Pandora's Box" model. He posited finitely many known sampling distributions with look-see costs; he proved an optimal index rule policy generalizing reservation wages. While elegant, Weitzman's model offered no uniform behavioral predictions, since the box payoff distributions were completely arbitrary.[6] *Our model with random additive known and hidden factors yields a tractable large random class of Weitzman models.*

Our model of search with learning[7] should also prove valuable across applied fields:

A. MACRO/LABOR ECONOMICS. McCall (1970) is the workhorse random search model, and yet despite assuming that all jobs are identical, it lacks predictions for distributional changes. Mortensen (1987) showed that job hunters' welfare rises with a mean-preserving spread of wages, but noted an ambiguous impact on unemployment duration. We find that unemployment increases in dispersion. Even when job seekers have prior job information, search duration increases as the prize distribution grows more dispersed, conditional on job characteristics (namely, our hidden factor). Also, we predict that those who have searched longer are more likely to stop.

A recent macro research question asks how search duration reacts to information technology improvements. Using CPS data, Kuhn and Skuterud (2004) find that web job search does not increase the job-finding rate, and Martellini and Menzio (2020) likewise finds a stable unemployment rate. They show that a more efficient match-

[6] Olszewski and Weber (2015) find a more general index rule; Doval (2018) lets Sam freely exercise unexplored options. Sam can explore old options or find new ones in Fershtman and Pavan (2022).

[7] Other work on optimal sequential search with learning include Rosenfield and Shapiro (1981); Ke and Villas-Boas (2019); Gossner et al. (2021); Nocke and Rey (2021).

ing function does not reduce the unemployment rate in the Diamond-Mortensen-Pissarides model. We model the nonstationary search process explicitly and so provide a clear microfoundation for their matching function. When firms receive more applications, and so have more options, we show that search duration rises.

B. Industrial Organization. Choi et al. (2018) use our two-factor search model to study price competition in an equilibrium application of ordered search models, where sellers post prices and buyers search. They reformulate price competition here as price competition in classic discrete choice models (Perloff and Salop, 1985). Our analysis fully characterizes the search, quitting and recall behavior.

C. Marketing. The workhorse consumer search model (e.g., Wolinsky, 1986; Anderson and Renault, 1999) assumes consumers who randomly and sequentially explore ex-ante identical goods. Recent work uses Weitzman's ordered search model (Armstrong, 2017). For instance, Kim et al. (2010) and Moraga-González et al. (2023) estimate it using search behavior data. With our model, these papers could analytically characterize search behavior, like search duration and recall.

D. Information Technology. Google search uses the PageRank Algorithm, and its PageRank score is like a known factor. A formal link is an open question.

The "click-through rate" is our option exploration decision, for which we make comparative static predictions. A nascent literature predicts online search behavior using statistical and machine learning techniques, e.g., (Zhou et al., 2018). In a similar spirit, we assume the modeler uses the searcher's past actions to predict the searcher's next action (continue, recall, strike or quit). While this literature typically assumes the searcher follows an exogenous decision rule, our searcher is maximizing.

After laying out the search model in §2, we derive Sam's optimal behavior in §3, and relate the value function slope to the eventual exercise chance. In §4, an outside observer predicts Sam's rising intensity of striking, recall, and quitting search. In §5, we show that prize dispersion lifts search duration, and explain how dispersion impacts our two factor search model. In §6, we explore variable accuracy search engines, and find an increasing returns to accuracy. In §7, we vary the number of options, finding e.g. that recall need not vanish with a vast number of options: Stationary search is thus a poor benchmark. An appendix contains all but the most instructive proofs.

2 A Two Factor Model of Search

A. General Model. A decision maker Sam sequentially searches through $N < \infty$ *inside options*. Each has random payoff $\mathcal{X} + \mathcal{Z}$, where $\mathcal{X}$ is the *known factor* and $\mathcal{Z}$ the *hidden factor*. Their respective densities g and h are log-concave, and thus their cdf's G and H are log-concave too. Each distribution has full support on $\mathbb{R}$, or an interval in $\mathbb{R}$ (Table 1). There is also a fixed *quit payoff* $u \in \mathbb{R} \cup \{-\infty\}$.[8]

The modeler faces *prospective uncertainty*: The known and hidden factors $\mathcal{X}$ and $\mathcal{Z}$ are independent random variables. On the other hand, Sam first learns all N realized known factors $\mathcal{X} = \chi$ before search. He cares about the ranked options $(\chi, \mathcal{Z})$, with random payoffs $\chi + \mathcal{Z}$, for varying known χ.

Sam faces a sequential search exercise, and seeks to maximize his expected payoff. While searching at stages $n = 0, 1, \ldots, N$, Sam may *explore* any inside option: To learn its realized hidden factor $\mathcal{Z} = z$, he pays a "look-see" or an expected *search cost* $c > 0$. This captures the mental or time toll of web search or list exploration. He may then either (i) *strike*, by *exercising* the current option, consuming its payoff and stopping search; or (ii) *pass*, by exploring a new inside option next stage; or (iii) *recall*, by exercising a previously passed option, or (iv) *quit*, by exercising the outside option. If Sam does not quit at $n = 0$, he *participates* and explores an inside option.

If Sam exercises an option with payoff w at stage $n \in \{0, 1, \ldots, N\}$, his final payoff is the value of the exercised option, less total search costs, or $w - nc$. *Stationary search* is the special case with a constant known factor $\mathcal{X}$ (degenerate, with point mass G).

With a constant known factor $\mathcal{X} \equiv \chi > u$, our model reduces to standard finite horizon search: Sam employs a constant cutoff, and recalls if he hits the last period. With a degenerate hidden factor $\mathcal{Z} \equiv 0$, Sam perfectly sorts options, taking the first.

We now give web search and partially informed search backstories for this model.

B. Web Search Special Case. The Gaussian distribution $\mathcal{W} \sim N(0, 1)$ is an especially useful log-concave distribution that captures search engines. For since a Gaussian r.v. is *stable*,[9] given a search query, the search engine parses Sam's payoff

[8] To be clear, this means that the ex post net economic value of an inside option is $\chi + z - u$.

[9] A r.v. X is *stable* if, for any iid copies X_1 and X_2, we have $aX_1 + bX_2$ equals $cX + d$ in distribution, $\forall a, b > 0$ and some $c > 0$ and $d \in \mathbb{R}$. Gaussian is the only finite variance stable distribution (Nolan, 2009), and so the only stable log-concave distribution (they have finite variance).

from each option into $W \equiv \mathcal{X} + \mathcal{Z}$ — where the known factor $\mathcal{X}$ is knowable and the hidden factor $\mathcal{Z}$ is unresolvable noise. So barring an omniscient search engine, $E[\mathcal{Z}^2] > 0$. The search engine lists web sites in descending order of known factors.

Write $\mathcal{X} \equiv \alpha X$ and $\mathcal{Z} \equiv \sqrt{1-\alpha^2} Z$, where X and Z are each independent and $N(0,1)$, by prospective independence. Then α is a suitable *accuracy* measure, for it transfers weight from the hidden to the known factor, as the search engine parses the option payoff W into

$$W = \alpha X + \sqrt{1-\alpha^2} Z. \tag{1}$$

C. Partially Informed Search Backstory. Our two-factor search model also captures prior information over options one faces — e.g. firms first view resumes of job applicants. Assume that Sam is endowed with an unbiased signal $\mathcal{X}$ of $\mathcal{W}$ from a quick synopsis, and a costly look-see resolves all remaining uncertainty $\mathcal{Z} = \mathcal{W} - \mathcal{X}$.

Our Gaussian accuracy model precisely captures this informed search story too. For assume inside options have Gaussian payoffs $W \sim N(0,1)$. Before searching, Sam observes a signal $X \sim N(\alpha w, 1-\alpha^2)$ for each option with true value w — say, a job advertisement. Upon seeing $X = x$, Sam updates his posterior beliefs to $W \sim N(\alpha x, 1-\alpha^2)$.[10] Since the noise in his estimate $Z = (W - \alpha x)/\sqrt{1-\alpha^2}$ is also $N(0,1)$, and is independent of X, the formula (1) arises with $X, Z \sim N(0,1)$. Notably, greater accuracy exactly corresponds to a more precise Gaussian signal.

3 Optimal Stopping Characterization

This section explores Sam's search problem for *realized known factors*: $\chi_1 \geq \cdots \geq \chi_N$.

A. Optimal Stopping. Consider two extreme cases. With a constant known factor, Sam samples the same distribution at most N times, recalling only at the last period.[11] With a degenerate hidden factor $\mathcal{Z} \equiv 0$, Sam stops at the first option, and so never recalls. With non-degenerate random known and hidden factors, Sam confronts a nontrivial nonstationary search problem, and recalls with positive probability.

By Lemma 1 below, Sam explores options in the rank order of known factors.

[10] Since X/α has mean w and precision $\alpha^2/(1-\alpha^2)$, the posterior precision of W is $1+\alpha^2/(1-\alpha^2) = 1/(1-\alpha^2)$ and its posterior mean is therefore $[(x/\alpha)\alpha^2/(1-\alpha^2)]/[1+\alpha^2/(1-\alpha^2)] = \alpha x$.

[11] The cutoff is the Weitzman index of each option (derived in (2)). His index formula yields the reservation wage formula in McCall's infinite horizon wage search model, embellished with recall.

Let F_n be the distribution of the random payoff $W_n = \chi_n + \mathcal{Z}_n$ for the option with known factor $\mathcal{X}_n = \chi_n$. Its realized payoff is $w_n = \chi_n + z_n$. Since $\chi_n \geq \chi_{n+1}$, the payoff cdfs $F_n(w) = H(w - \chi_n)$ fall in n in the sense of first-order stochastic dominance. The *reservation prize* $\bar{w}_n$ for entering stage n obeys Weitzman's indifference equation:

$$\bar{w}_n = -c + \bar{w}_n F_n(\bar{w}_n) + \int_{\bar{w}_n}^{\infty} w dF_n(w). \tag{2}$$

Log-concavity of H ensures finite moments: $\bar{w}_n < \infty$ for all n. Integration by parts yields $c = \int_{\bar{w}_n}^{\infty} 1 - F_n(z) dz$, as is standard in search theory. Because F_n stochastically falls in n, so too do reservation prizes, namely, (★): $\bar{w}_1 \geq \cdots \geq \bar{w}_N$.

Given realized inside option payoffs $w_1, w_2, \ldots, w_N$, the dynamic programming state variable for stages $n = 1, \ldots, N$ are *fallback* payoffs $\Omega_n = \max(u, w_1, \ldots, w_n)$ with $\Omega_0 = u$. By Weitzman (1979), Sam explores options in order of known factors.

Lemma 1 (Optimal Search) *Sam explores new options in the falling order* (★) *of reservation prizes. In stage n, he stops searching when $\Omega_n \geq \bar{w}_{n+1}$. Specifically, he strikes if $w_n \geq \max\{\bar{w}_{n+1}, \Omega_{n-1}\}$, and recalls any fallback if $\Omega_{n-1} \geq \max\{\bar{w}_{n+1}, w_n\}$.*

Intuitively, Sam strikes if the present option beats the past and the future, or $w_n \geq \max(\bar{w}_{n+1}, \Omega_{n-1})$. He quits / recalls if the past beats the present and future, or $\Omega_{n-1} \geq \max(\bar{w}_{n+1}, w_n)$. He passes to the next option if the future beats the past and present, or $\bar{w}_{n+1} > \Omega_n$. This triple choice is new in nonstationary sequential search.

Let the search *optionality value* $\zeta(c)$ be the reservation wage in stationary wage search with a zero known factor. As is well-known, this solves the discrete first order condition (FOC):

$$c = \int_{\zeta(c)}^{\infty} [1 - H(z)] dz. \tag{3}$$

In our two factor model, $\zeta(c)$ captures upside benefits of the random hidden factor $\mathcal{Z}$.

Lemma 2 (Reservation Prizes) *In stage $n-1$, Sam accepts $w \geq \bar{w}_n = \chi_n + \zeta(c)$.*

This expression follows from integrating the tail integral (2) by parts, using (3):

$$c = \int_{\bar{w}_n}^{\infty} [1 - F_n(w)] dw = \int_{\bar{w}_n}^{\infty} [1 - H(w - \chi_n)] dw = \int_{\bar{w}_n - \chi_n}^{\infty} [1 - H(z)] dz.$$

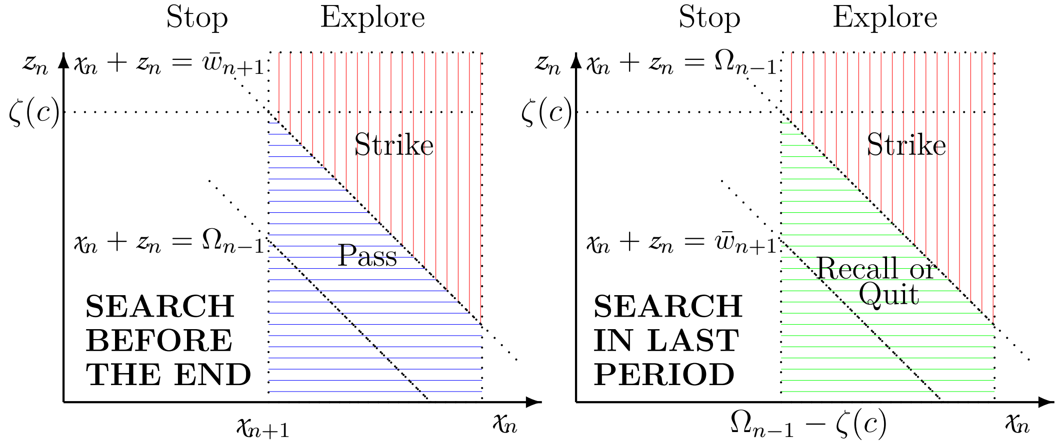

Figure 1: **The Phase Transition in Search.** We plot Sam's optimal behavior given the known and idiosyncratic factors, χ and z. Until a phase transition, $\bar{w}_{n+1} > \Omega_{n-1}$, and Sam's choice is always between strike and pass (left). We then transition to $\Omega_{n-1} \geq \bar{w}_{n+1}$, whereupon Sam's decision margin shifts to strike or recall / quit (right).

The fallback Ω_n rises in n, while the reservation value $\bar{w}_n$ falls in n, by Lemma 1. Sam's future is initially brighter than his past, $\bar{w}_{n+1} \geq \Omega_{n-1}$, and he either passes or strikes. But in Figure 1, the $\chi_n + z_n = \bar{w}_{n+1}$ line shifts left each stage; there comes a *recall moment*, after which $\Omega_{n-1} \geq \bar{w}_{n+1}$, when Sam strikes or quits/recalls.

Lemma 3 (Recall Moment) *Sam's choice shifts from strike or pass, to strike or quit/recall, ending search. He stops sooner for a higher search cost c or quit payoff u.*

Proof: As Sam either strikes or passes in stage n when $\Omega_{n-1} < x_{n+1} + \zeta(c)$, the recall moment is at least n with probability $P(\max_{j \leq n-1}\{u, x_j + \mathcal{Z}_j\} < x_{n+1} + \zeta(c))$. This chance falls in c and u, and so the transition time falls stochastically in c and u. □

B. Value Functions. The *value function* $V_n(\Omega_n)$ at stage n is the maximum payoff when the best option so far is Ω_n. Clearly, $V_N(\Omega_N) = \Omega_N$. For any $n < N$, backward induction yields value functions $V_{n-1}, \ldots, V_1$ via the Bellman equation:

$$V_n(\Omega_n) = \max\left\{\Omega_n, -c + V_{n+1}(\Omega_n)F_{n+1}(\Omega_n) + \int_{\Omega_n}^{\infty} V_{n+1}(z)dF_{n+1}(z)\right\}. \quad (4)$$

Sam optimally stops at stage n seeing only the next known factor x_{n+1}; this is the dynamic programming one-stage look-ahead property for optimal search. For the reservation prize $\bar{w}_{n+1}$ depends only on F_{n+1} in (2).

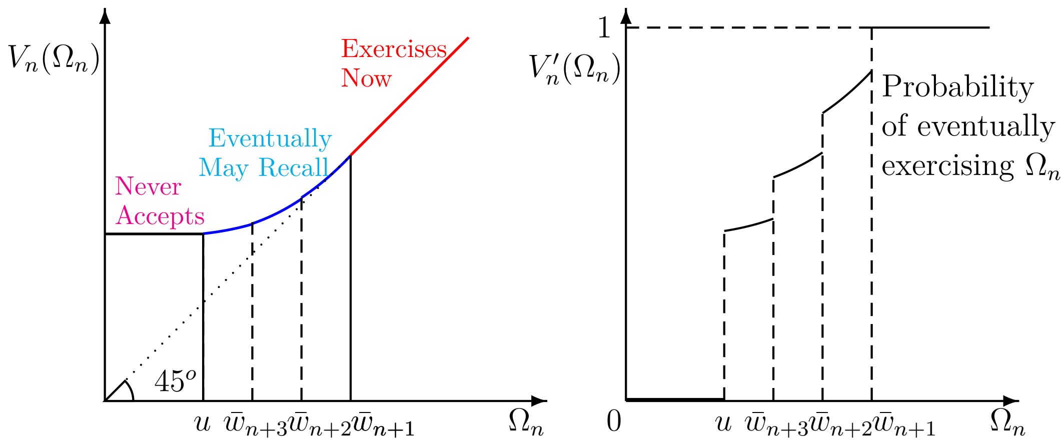

Figure 2: **The Value Function Slope is the Eventual Exercise Chance.** To depict Lemma 4, we schematically plot a value V_n and its slope in the fallback $\Omega_n = \max(w_1, \dots, w_n)$. Sam immediately exercises a prize $w \geq \bar{w}_{n+1}$, and recalls Ω_{n-1} if $n \geq 2$ iff $\Omega_{n-1} = \Omega_n \in [\bar{w}_{n+1}, \bar{w}_n)$. So V_n is constant on $(-\infty, u)$, then increasing and strictly convex on $[u, \bar{w}_{n+1})$, and finally on the diagonal — and its slope is first 0, then positive and increasing, and eventually one (at right). That *the Bellman value slope is the eventual exercise chance is a defining feature of this search model.*

The reservation wage is the continuation value in stationary search. But here Sam's reservation prize exceeds his continuation value: $\bar{w}_{n+1} > V_n(\Omega_n)$ for $\Omega_n < \bar{w}_{n+1}$, because the known factors χ_n fall in n. As his fallback Ω_n improves, Sam's value V_n increases on $(u, \bar{w}_{n+1})$, as Figure 2 depicts.

Lemma 4 (Value Slope) *$V_n(\Omega_n)$ is convex in Ω_n in stages $n = 1, \dots, N$, and $V_n'(\Omega_n)$ exists $(\bar{w}_{n+1}, \bar{w}_n)$, and is Sam's chance of eventually exercising the fallback Ω_n. As c rises, the slope $V_n'(\Omega)$ weakly rises, as does the eventual exercise chance.*

The stationary search value function is flat until the reservation prize, and then is the 45^o line — so the slope is the eventual (and in fact immediate) acceptance chance. The rising marginal value here reflects the rising chance of exercising the fallback option (but here only by recall). This slope interpretation holds in the last period here, since V_N is just the 45^o line. Inductively, assume Sam eventually exercises the fallback option Ω_n at stage $n+1$ with chance $V_{n+1}'(\Omega_n)$. Differentiating the Bellman equation (4) for fallbacks $\Omega_n < \bar{w}_{n+1}$ yields $V_n'(\Omega_n) = F_{n+1}(\Omega_n)V_{n+1}'(\Omega_n)$. By Lemma 1, at stage n, Sam eventually exercises the fallback option Ω_n if it is the best in all stages $k \geq n$. By independence of hidden factors across stages,

$P(\Omega_n \text{ best in stages } k \geq n) = P(\Omega_n \text{ best in stage } n+1) P(\Omega_n \text{ best in stages } k \geq n+1).$

Given this equality, and $V'_N \equiv 1$, the probabilistic meaning of V'_n follows by induction.

4 How Does Search Change Over Time?

We now see how search monotonically change over time. We characterize the respective stage n *stopping*, *quitting*, *recalling*, and *exercising chances* — namely, $\mathcal{S}_n$, $\mathcal{Q}_n$, $\mathcal{R}_n$, and $\mathcal{E}_n$ — for a modeler *unable to see the known factors*. Then $\mathcal{S}_n = \mathcal{Q}_n + \mathcal{E}_n$, since Sam either quits or exercises an inside option after stopping. In a stationary search model, behavior is constant over time, so that $\mathcal{Q}_n = \mathcal{R}_n = 0$, $\mathcal{S}_n = \mathcal{E}_n$ is invariant to n.

A. Search Survival Chances. We first develop a simple probabilistic building block to reflect the modeler's thinking in this nonstationary world. Assume two inside options A and B. Say that A *delays* B if Sam optimally explores A and then B. A random option $(\mathcal{X}, \mathcal{Z})$ delays one with known factor $\hat{\chi}$ if $(\mathcal{X}, \mathcal{Z})$ has a reservation prize above $\hat{\chi}$, but a realized prize below $\hat{\chi} + \zeta(c)$. This has *delay chance* equal to

$$\delta(\hat{\chi}, c) \equiv P\left(\{\mathcal{X} > \hat{\chi}\} \cap \{\mathcal{X} + \mathcal{Z} < \hat{\chi} + \zeta(c)\}\right) = \int_{\hat{\chi}}^{\infty} H\left(\hat{\chi} + \zeta(c) - x\right) g(x) dx. \quad (5)$$

The *participation chance* σ_1 is Sam's chance of starting search. Obviously, $\sigma_1 = 1$ given a quit payoff $u = -\infty$, but otherwise, it is nontrivial. More generally, the *n-th survival chance* σ_n is the chance that Sam's search lasts at least n stages — unconditional on known factors. Easily, $\sigma_0 = 1 > 0 = \sigma_{N+1}$. Then σ_n is the chance that

$$\text{Sam is willing to explore the option } (\chi_n, \mathcal{Z}_n)\text{, i.e., } \chi_n + \zeta(c) > u \quad (6a)$$

$$n-1 \text{ options } (\mathcal{X}', \mathcal{Z}') \text{ delay option } n\text{, i.e., } \mathcal{X}' + \zeta(c) > \chi_n + \zeta(c) > \mathcal{X}' + \mathcal{Z}' \quad (6b)$$

$$\text{the other } N-n \text{ options } (\mathcal{X}', \mathcal{Z}') \text{ have known factors below } \chi_n \quad (6c)$$

Events (6b) and (6c) have chances $\delta(\chi_n, c)^{n-1}$ and $G(\chi_n)^{N-n}$. Integrating the binomial probability of (6a)–(6c) over known factors χ_n of all options, prospective independence yields

$$\sigma_n = N \binom{N-1}{n-1} \int_{u-\zeta(c)}^{\infty} \delta(\chi_n, c)^{n-1} G(\chi_n)^{N-n} g(\chi_n) d\chi_n. \quad (7)$$

Then the survival chance σ_n falls in the search cost c and the quit payoff u.[12]

B. Selection Effects. Predicting Sam's behavior is thorny as he is apprised of known factors, but his continued search signals higher known factors. While search concludes for given known factors (Lemma 3), this "selection effect" makes Sam less likely to stop the longer he searches. We argue that log-concavity precludes a perverse search duration jump. To see this, consider for a moment two options, with the first known. Then the expected gap $E[\mathcal{X}_1 - \mathcal{X}_2|\mathcal{X}_1 = x_1]$ rises in x_1 if G is log-concave.[13] In other words, Sam enters stage 2 less often with a higher known factor x_1.

Theorem 1 (Search Intensifies) *Sam's conditional recall and exercise chances, $\mathcal{R}_n$ and $\mathcal{E}_n$, rise in the stage n, as does the quitting chance $\mathcal{Q}_n$ for small costs $c>0$.*

For intuition into this result for the stopping chance $\mathcal{S}_n$, assume the modeler sees all past known factors, and that $-\mathcal{X} \sim \exp(\lambda)$, and Sam never quits ($u = -\infty$). We argue that the log-concavity of the hidden factor cdf H blunts the discussed selection effects. As Sam never quits, the exercise and stopping chance coincide, and it suffices that $\mathcal{E}_n = \mathcal{S}_n$ rises in n. Define the order statistic gap $\Delta_j \equiv x_j - \mathcal{X}_{j+1}$. Then:

$$\mathcal{S}_n = 1 - \frac{P(\text{Explores option } n+1)}{P(\text{Explores option } n)} = 1 - \frac{P(x_j + \mathcal{Z}_j < \mathcal{X}_{n+1} + \zeta, \forall j = 1, ..., n)}{P(x_j + \mathcal{Z}_j < x_n + \zeta, \forall j = 1, ..., n-1)}$$
$$= 1 - \frac{E[H(\zeta - \Delta_n)\Pi_{j=1}^{n-1} H(\zeta - x_j + x_n - \Delta_n)]}{\Pi_{j=1}^{n-1} H(\zeta - x_j + x_n)}$$

where the expectation is over realized gap Δ_n. By the memoryless property of the exponential distribution, Δ_j has distribution $\exp(\lambda(N-j))$, and is independent of the observed known factors $(x_1, ..., x_j)$.[14] Now, $\mathcal{S}_{n+1} > \mathcal{S}_n$ because Δ_{n+1} stochastically dominates Δ_n, and because the ratios (8) rise in n, as H is log-concave and $x_n > x_{n+1}$:

$$\frac{H(\zeta - x_j + x_n - \Delta_n)}{H(\zeta - x_j + x_n)} = P(x_j + \mathcal{Z}_j < \mathcal{X}_{n+1} + \zeta | x_j + \mathcal{Z}_j < x_n + \zeta). \tag{8}$$

[12]The lower bound $u - \zeta(c)$ of the integral (7) rises in c by (3), and the delay chance $\delta(x, c)$ falls in c by (3)–(5). Easily, *the survival chance σ_n falls in the outside option payoff u*, as $u - \zeta(c)$ rises.

[13]Let $\mathcal{X}_{(-\infty,a]}$ be the right truncation of the r.v. $\mathcal{X}$ at a. By Theorem 3.B.19 in Shaked and Shanthikumar (2007), if G is log-concave, then $\mathcal{X}_{(-\infty,a]}$ grows more *dispersive* as a rises: Every pair of quantiles of $\mathcal{X}_{(-\infty,a]}$ push further from each other (see (10)). So $E[\mathcal{X}_1 - \mathcal{X}_2|\mathcal{X}_1 = x_1]$ rises in x_1.

[14]Theorem 3.B.19 in Shaked and Shanthikumar (2007) posit $1 - G$ log-concave. In the log-linear case: *if $\mathcal{X}$ has an exponential distribution with mean λ, the order statistic gap $\mathcal{X}_n^N - \mathcal{X}_{n+1}^N$ is exponentially distributed with mean $n\lambda$, for all N* (Pyke, 1965). So hazard rates are constant in N.

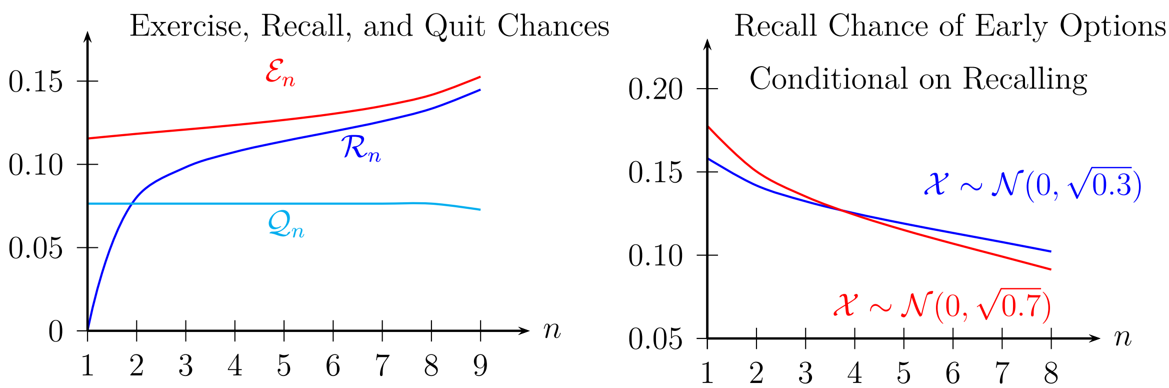

Figure 3: **Intensifying Search Over Time.** Left: By Theorem 1, the hazard rates of recall $\mathcal{R}_n$ and exercising an inside option $\mathcal{E}_n \equiv \mathcal{R}_n + \mathcal{K}_n$ rise in n (here $\mathcal{X} \sim \Gamma(1.2, 2)$, $\mathcal{Z} \sim \Gamma(2.8, 2)$, $c = 0.2$, $N = 10$, $u = 0$). Right: By Theorem 2, Sam recalls earlier options more often, and so recall probabilities fall in n — from stage 9. The chance of recalling the earliest options rises in $\mathcal{X}$ dispersion (now with $c = 0.1$ and $u = -10$).

We now answer which option Sam recalls most. Earlier options have larger known factors, and have been passed over more often; this offers more damning selection evidence of their hidden factors. Surprisingly, these options are most sought after:

Theorem 2 (Older Options Recalled More Often) *If Sam explores option n, then the chance that he recalls any prior option $j < n$ falls in j, for all $n = 3, \ldots, N$.*

Proof: If Sam explores option n, his payoff from any prior option falls below the cutoff $\bar{w}_n = \chi_n + \zeta(c)$, or search would have stopped. By the Markov property of order statistics,[15] the joint distribution of known and hidden factors for the first $n-1$ options is that of $n-1$ i.i.d. draws $(\mathcal{X}, \mathcal{Z})$ from (G, H), *conditional* on the known factor $\mathcal{X} > \chi_n$ and the selection effect $\mathcal{X} + \mathcal{Z} < \chi_n + \zeta(c)$. If $\mathcal{X} = \chi > \chi_n$ is the realized known factor of any prior option, its payoff $W \equiv \chi + \mathcal{Z}$ so has the cdf:

$$P(W \leq w | W < \chi_n + \zeta(c)) = \frac{H(w - \chi)}{H(\chi_n + \zeta(c) - \chi)}. \tag{9}$$

As $w < \chi_n + \zeta(c)$, this cdf of W falls in χ by log-concavity of H, and so W stochastically increases in χ — namely, the payoffs of earlier options are stochastically ranked. As

[15] Let $X_{1:n} \geq X_{2:n} \geq \cdots \geq X_{n:n}$ be order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a population with cdf F and pdf f. Given $X_{i:n} = x_i$, the distribution of $X_{j:n}$, for $j < i$, is the same as that of the j-th order statistic of an $(n-i)$ sample from a population with distribution F truncated at the left by x_i. See Theorem 2.4.1 in Arnold et al. (1992).

this ordering holds for all $\mathcal{X}_n$ realizations, it holds unconditional on $\mathcal{X}_n$. □

The premise of Theorem 2 is tight: *later options are recalled more often with a log-convex hidden factor cdf.* For then (9) rises in χ: recent options are stochastically better. At the knife-edge of H exponential, the recall chance is constant. Truncation in (9) favors later options with logconvexity, and earlier ones with log-concavity.

5 Stochastic Changes that Raise Search Duration

We now ask what distribution changes lift *search duration*, or the mean search time. After resolving this open question in stationary search, we then adapt the solution for our model.[16] This question is important: What wage distribution shifts raise unemployment? If inflation impacts the price distribution, do consumers search more?

The duration results here don't need log-concavity assumptions. When a prize distribution incurs a mean preserving spread (MPS), search duration may rise or fall:[17] For while the reservation wage rises, the stopping probability (area above it) may rise or fall (Figure 4).[18] The problem is that a MPS allows localized distributional compression, as in the left panel of Figure 4. In its right panel, the distribution scales $\mathcal{Z} \mapsto a\mathcal{Z}$, for $a > 1$. This depicts the dispersion stochastic order: $\mathcal{Z}_B$ is *more disperse* than $\mathcal{Z}_A$ if any two quantiles of the cdf H_B are further apart than those of H_A, i.e.

$$H_B^{-1}(\alpha'') - H_B^{-1}(\alpha') \geq H_A^{-1}(\alpha'') - H_A^{-1}(\alpha') \qquad \text{for all } 0 < \alpha' \leq \alpha'' < 1. \tag{10}$$

As the quantile function H_B^{-1} is steeper than H_A^{-1}, and each is differentiable, the densities rank oppositely: $h_B(H_B^{-1}(\alpha)) \leq h_A(H_A^{-1}(\alpha))$ for all $\alpha \in (0, 1)$.[19] Parametric

[16] In a study of risk, Chateauneuf et al. (2004) have a no discounting example of stationary search, in which duration rises if the reward distribution grows *location independent riskier* (Jewitt, 1989).

[17] Notably, Keane et al. (2011) claimed that search duration rises in a MPS. For a simplest possible counterexample, let $P(\mathcal{Z}_A = 1) = P(\mathcal{Z}_A = 2) = P(\mathcal{Z}_A = 3) = 1/3$. If $c = 1/4$, then $\zeta_A(c) = 3$ by (3). Search ends with chance $P(\mathcal{Z}_A \geq 3) = 1/3$. Spread $\mathcal{Z}_A$ to $\mathcal{Z}_B$, where $P(\mathcal{Z}_B = 1) = P(\mathcal{Z}_B = 3) = 1/2$. Now, $\zeta_B(c) = 3$ by (3), and $P(\mathcal{Z}_B \geq 3) = 1/2$. Search ends with a higher chance. But if $c = 1/2$, then $\zeta_A(c) = 2$ and $P(\mathcal{Z}_A \geq 2) = 2/3$, while $\zeta_B(c) = 3$ and $P(\mathcal{Z}_B \geq 3) = 1/2$. Search duration increases. See here for another counterexample with the Pareto distribution.

[18] We assume h is symmetric around 0 and follows the Weibull distribution on each side, namely $h(z) = (\alpha/\beta)[(|z| - \mu)/\beta]^{\alpha-1} e^{-[(|z|-\mu)/\beta]^{\alpha}}$ for $|z| > \mu$ and 0 otherwise. The parameters for the blue line are $\{\alpha, \beta, \mu\} = \{1, 0.2, 0\}$ and that for the orange line are $\{\alpha, \beta, \mu\} = \{2, 0.2, 0.5\}$ in the left panel and $\{\alpha, \beta, \mu\} = \{1, 0.4, 0\}$ in the right panel. The search cost is $c = 0.05$.

[19] Then cdf's cross once — for when they coincide, H_A increases faster than H_B.

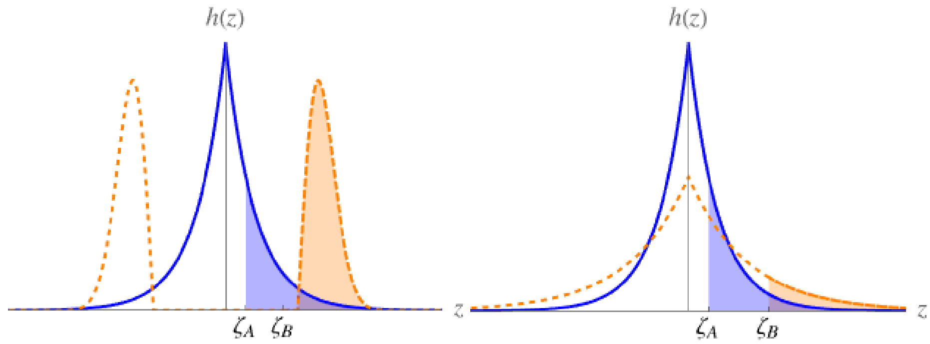

Figure 4: **Search Duration Can Rise or Fall in a Mean-Preserving Spread.** We plot the probability density $h(z)$. The shaded area indicates the stopping chance, with dashed orange a MPS of the solid blue. The stopping chance (above ζ_A, ζ_B) rises in the left panel, but falls in the right. The orange dashed stopping area at left is exactly one-half, exceeding the blue solid; the reverse is true in the right panel.

changes induce the dispersion order in many common distributions (see Table 1).[20,21]

Search duration is the reciprocal of the *stopping probability* $S(c) = 1 - H(\zeta(c))$. Change variables in the Bellman equation (3) from the prize z to its cdf $\alpha = H(z)$. The *quantile function* inverse $z = H^{-1}(\alpha)$ obeys $dz = dH^{-1}(\alpha) = [\partial H^{-1}(\alpha)/\partial\alpha]d\alpha$. So (3) gives:

$$\int_{1-S(c)}^{1} (1-\alpha)\frac{\partial H^{-1}(\alpha)}{\partial\alpha}d\alpha = \int_{\zeta(c)}^{\infty}[1-H(z)]dz = c. \tag{11}$$

In other words, search cost equals the expected survivor probability w.r.t. the quantile distribution. By (11), the stopping probability $S(c)$ falls if the quantile function H^{-1} everywhere grows steeper, and the slope $\partial H^{-1}(\alpha)/\partial\alpha$ everywhere rises (Figure 5).

Theorem 3 (Stationary Search) *Search duration rises in dispersion if* $\mathcal{X} \equiv 0$.

Our model is undiscounted. In the stationary *discounted* wage search model of McCall (1970), duration *falls in dispersion for low search costs* (see here).

When dispersion rises, Sam searches more if his reservation prize ζ rise swamps the effect of higher tail probability. Smoothly index the prize pdf h_t and cdf H_t of $\mathcal{Z}$ by a dispersion index $t \in \mathbb{R}$. Inspired by consumer theory, we now decompose the

[20]Unlike stochastic dominance, the dispersion order is location free, e.g. $N(\mu, \sigma^2)$ has the same dispersion for all μ. But a mean-preserving increase in dispersion implies a mean-preserving spread. See Shaked and Shanthikumar (2007) (SS) for a thorough review.

[21]Ganuza and Penalva (2010) use the dispersive order to study information disclosure in auctions, and Zhou (2017) for price orders in bundling. Dispersion can be seen as iff for Theorem 3. See here.

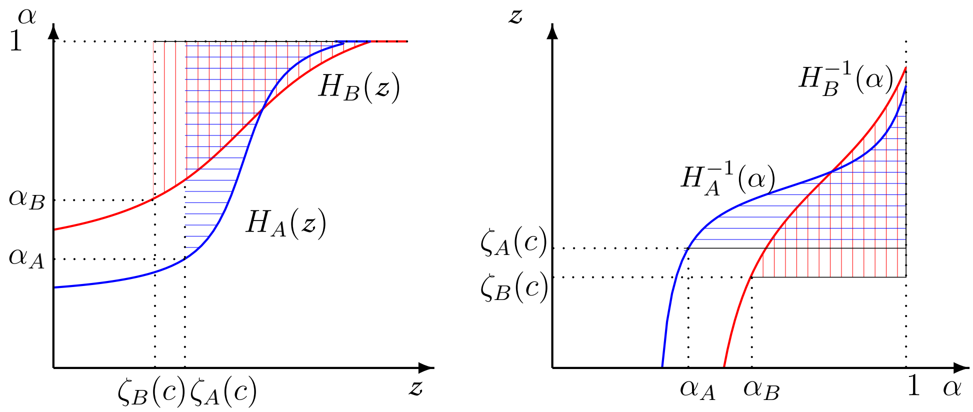

Figure 5: **Graphical Proof that Search Duration Rises in Prize Dispersion**. If H_B is more dispersed than H_A, its inverse quantile plot H_B^{-1} is steeper than H_A^{-1} (at right). By (11), the area above a cdf H right of its cutoff $\zeta(c)$ equals the search cost c. So the (shaded) quantile slope weighted survivor integral in the right panel is greater for any cutoff z. By (11), the cutoffs are ranked $\zeta_A(c) > \zeta_B(c)$ and the conditional stopping chance is higher: $\alpha_B = H_B(\zeta_B(c)) > H_A(\zeta_A(c)) = \alpha_A$.

stopping chance change $\frac{d}{dt}[1 - H_t(\zeta_t)]$ into the sum of a *substitution effect* and *direct effect*. Differentiate the reservation prize equation $\int_{\zeta_t}^{\infty}[1 - H_t(z)]dz \equiv c$ from (3) in t. Then[22] $-[1 - H_t(\zeta_t)]\dot{\zeta}_t = \int_{\zeta_t}^{\infty} \dot{H}_t(z)dz$, which affords us the decomposition:

$$\frac{d[1 - H_t(\zeta_t)]}{dt} = -h_t(\zeta_t)\dot{\zeta}_t - \dot{H}_t(\zeta_t) = \frac{h_t(\zeta_t)}{1 - H_t(\zeta_t)} \int_{\zeta_t}^{\infty} \dot{H}_t(z)dz - \dot{H}_t(\zeta_t) \qquad (12)$$

Assume the quantile function H_t^{-1} smoothly steepens in t, in the dispersive order. Since $\partial H_t^{-1}(a)/\partial a$ rises in t, $\partial H_t^{-1}(a)/\partial t$ rises in $a = H_t(z)$. Then the derivative[23] $\partial H_t^{-1}(a)/\partial t = -\dot{H}_t(z)/h_t(z)$ strictly rises in z. Adjusting a ratio inequality exploited in Smith (2006),[24] we have $\int_{\zeta_t}^{\infty} \dot{H}_t(z)dz / \int_{\zeta_t}^{\infty} h_t(z)dz < \dot{H}_t(\zeta_t)/h_t(\zeta_t)$. So the stopping chance $1 - H_t(\zeta_t(c))$ falls in t since the derivative (12) is negative. *The substitution effect swamps the direct effect for prize dispersion* — the first term of (12) dominates.

In our two factor search model, greater dispersion of the hidden factor alone will not raise search duration given the dual payoff source. Since exploring an option with a known factor χ pays more than quitting when $\chi + \zeta(c) > u$, we need that $\chi + \zeta(c)$

[22] We use Newton's notation $\dot{x}$ for t derivatives of any function x.

[23] This equality follows from $0 = \frac{\partial}{\partial t}a = \frac{\partial}{\partial t}H_t(H_t^{-1}(a)) = \dot{H}_t(H_t^{-1}(a)) + h_t(H_t^{-1}(a))\partial H_t^{-1}(a)/\partial t$.

[24] If $b(z) > 0$, with $a(z)/b(z)$ decreasing, then $(\int_y a(z)dz)/(\int_y b(z)dz) < a(y)/b(y)$. Smith cites "a special case of a continuous variable generalization of inequality 3.3.15 in Mitrinović (1970)".

Distribution	cdf	Support	More Disperse if	Thin tail?
Exponential	$1-e^{\lambda z}$	$[0,\infty)$	$\lambda\uparrow$	No
Gamma	$\frac{1}{\Gamma(k)}\gamma(k,z/\theta)$	$[0,\infty)$	$\theta\uparrow$	No
Gaussian	$\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{z-\mu}{\sigma\sqrt{2}}\right)$	$(-\infty,\infty)$	$\sigma\uparrow$	Yes
Gumbel	$e^{-e^{-(z-\mu)/\beta}}$	$(-\infty,\infty)$	$\beta\uparrow$	No
Logistic	$1/\left(1+e^{-\frac{z-\mu}{s}}\right)$	$(-\infty,\infty)$	$s\uparrow$	No
Uniform	$(z-a)/(b-a)$	$[a,b]$	$a\downarrow$ or $b\uparrow$	Yes

Table 1: **Dispersion Examples.** Logconcave distributions are often parameterized by dispersion. The last column flags if recall chances vanish as $N\to\infty$ (Theorem 10).

not fall as $\mathcal{Z}$ grows more disperse.[25] If $\mathcal{Z}_B$ is more disperse than $\mathcal{Z}_A$, call $\mathcal{Z}_B$ a *mean-enhancing dispersion* of $\mathcal{Z}_A$ if $E[\mathcal{Z}_B]\geq E[\mathcal{Z}_A]$. In this case, the search optionality values rank $\zeta_B(c)\geq\zeta_A(c)$,[26] and we can derive increased search duration in §C.

Theorem 4 (Hidden Factor Dispersion) *After a mean enhancing dispersion in the hidden factor $\mathcal{Z}$, survival chances σ_n rise, as does the participation chance σ_1 and search duration. The recall moment is later with a mean-preserving dispersion for $\mathcal{Z}$.*

Search duration rises given increased survival chances, owing to $\tau=\sum_{n=1}^{N}\sigma_n$.

In our Gaussian factors web search model in §2, hidden factor dispersion rises iff the known factor dispersion falls, by equation (1). But in our more general model where these variances need not trade off, we argue that greater known factor dispersion shortens search. For order statistics $\{\mathcal{X}_n\}$ drop faster, and so Sam stops sooner, because exploring the next option is a less inviting prospect. More precisely:

Theorem 5 (Known Factor Dispersion) *Assume quit payoff $u=-\infty$. If known factor dispersion rises, then every survival chance σ_n falls, as does the search duration.*

For insight, assume a vanishing known factor dispersion, with a finite horizon pure stationary search model. Then a constant reservation prize $\zeta(c)$ emerges, by Lemma 2.

[25] Dispersion in $\mathcal{Z}$ can reduce search duration via raising the quitting chance. In here we show that a more dispersed hidden factor accelerates quitting iff the quit payoff is low.

[26] By the dispersion order, H_B^{-1} is steeper than H_A^{-1}. If H_A and H_B have the same mean, H_A single crosses H_B, and so $\mathcal{Z}_B$ is a MPS of $\mathcal{Z}_A$ (Diamond and Stiglitz, 1974). Then $\zeta_B(c)\geq\zeta_A(c)$ by (3). If $\mathcal{Z}_B$ has a higher mean than $\mathcal{Z}_A$, then $\mathcal{Z}_B$ is a mean-preserving increase in dispersion of $\mathcal{Z}_A$ and a right distribution shift, as the dispersion order is location free. Each shift lifts $\zeta_B(c)$ above $\zeta_B(c)$.

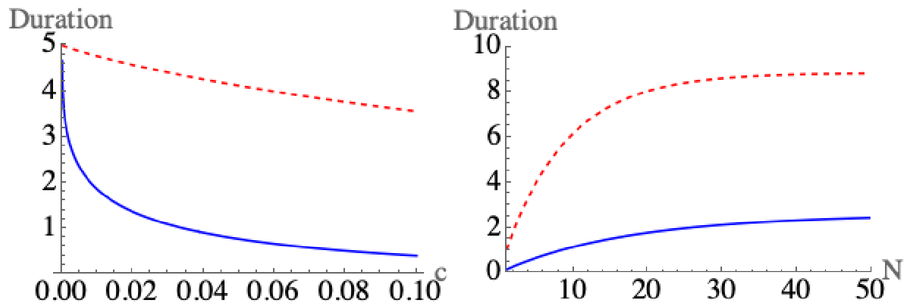

Figure 6: **Search With and Without Prior Information** (solid vs. dashed lines). Payoffs are the sum of a zero mean known factor of standard deviation 0.44, and a zero mean, unit variance hidden factor. The quit payoff is 0.78. The search cost changes at left (fixing $N = 5$ options), and the number of options varies at right (for search cost \$0.06). This Monte Carlo simulation uses (7), and depicts Corollary 1: search duration drops given prior information, for a low enough quit payoff.

Otherwise, the known factor $\mathcal{X}_n$ has the cdf of $G^{-1}(U_n)$, for the nth uniform $[0,1]$ order statistic U_n, the *order statistic gap* $\mathcal{X}_n - \mathcal{X}_{n+1} \sim G^{-1}(U_n) - G^{-1}(U_{n+1})$ increases stochastically in the dispersion of $\mathcal{X}$ by (10), reducing the survival chances σ_n. Intuitively, consecutive options are expected to fall more in expected payoff with a more disperse known factor. But since dispersion raises tail weight of $\mathcal{X}$, it might lift Sam's participation chance, when $\mathcal{X}_1 + \zeta(c) \geq u$, and thereby increase σ_n. This countervailing participation effect obviously vanishes with no outside option, $u = -\infty$.

We are now equipped to answer a misspecification question. If researchers ignore Sam's prior information, and treat a two factor model as stationary, with known factor $\mathcal{X} = 0$, then they over-predict search duration. If $\mathcal{X} = 0$, with a quit payoff $u \geq \zeta(c)$, then Sam never searches, and more known factor dispersion raises duration. If $u < \zeta(c)$, by Theorem 5's proof, search ends sooner with a dispersed known factor:

Corollary 1 *If $u < \zeta(c)$, Sam searches, but less than with a degenerate known factor.*

In here, we calibrate parameters to match the search duration, purchase chance, and recall chance in the online book market studied in De Los Santos et al. (2012). In their data, among the transactions in which consumers visited more than one online book store, around 40% recalled a prior option. Our non-stationary sequential search model better explains the data than a stationary model.

In Figure 6, we depicts how the stationary search model over-predicts duration for their data. Pre-search information greatly reduces search duration, as is intuitive.

6 Search Outcomes and Web Search Accuracy

6.1 How Accuracy Impacts Search Duration and Outcomes

We consider how a search engine of varying accuracy impacts search. Recall the Gaussian web search application of §2B, with payoff (1). The extremes are standard sequential search with $\alpha = 0$, and a given ranking of all known factors $X_1 > X_2 > \cdots$. Search optionality $\zeta(\alpha, c)$ in (3) solves for the Gaussian cdf Φ:

$$c = \int_{\zeta(\alpha,c)}^{\infty} \left[1 - \Phi\left(\frac{s}{\sqrt{1-\alpha^2}}\right)\right] ds \tag{13}$$

As α rises, the known factor grows more disperse, and the hidden factor less so.

We show that typical search engine measures do not flag accuracy. A common search engine ranking tool is the *click through rate* (CTR), or the chance that a visitor explores a link after posting a query.[27] This is the chance Sam explores the first option after seeing all known factors, i.e. $\sigma_1 = P(\mathcal{X}_1 > \ell(\alpha, u, c))$. Next, the *quitting chance* q is the chance Sam does or does not search, but eventually chooses the outside option.

By Lemma 2, Sam searches iff the highest known factor is $X_1 > \ell(\alpha, u, c)$, where

$$\ell(\alpha, u, c) \equiv \frac{u - \zeta(\alpha, c)}{\alpha}. \tag{14}$$

Our first case posits $u = -\infty$ to ensure Sam searches. Since he also never quits, his optimal payoff $\mathcal{V}(\alpha, c)$ equals the mean *accepted option* $\mathcal{W}^A(\alpha, c)$ less expected search costs $c\tau(\alpha, c)$:

$$\mathcal{V}(\alpha, c) = E[\mathcal{W}^A(\alpha, c)] - c\tau(\alpha, c). \tag{15}$$

By Blackwell's Theorem, the optimal value $\mathcal{V}(\alpha, c)$ rises in α. With $u = -\infty$, this happens by lower search duration $\tau(\alpha, c)$ or greater accepted option $E[\mathcal{W}^A(\alpha, c)]$.[28]

Theorem 6 (Rising Accuracy, I) *Assume a quit payoff $u = -\infty$. The mean accepted option $E[\mathcal{W}^A(\alpha, c)]$ obeys $\frac{\partial}{\partial \alpha} E[\mathcal{W}^A(\alpha, c)] < 0$ for small $c > 0$, and all $\alpha \in (0, 1)$. Search duration $\tau(\alpha, c)$ monotonically falls in accuracy α, for all $\alpha \in (0, 1)$.*

[27] This is a commercial measure of search engines, as clicking means that search ads are seen.

[28] From §2, greater accuracy implies more informative search engine signal in Blackwell's sense: Shifting to accuracy α_L from $\alpha_H > \alpha_L$ adds zero mean Gaussian noise with variance $\alpha_H^2 - \alpha_L^2$ to the hidden factor, and raises its variance to $(1 - \alpha_H^2) + (\alpha_H^2 - \alpha_L^2) = 1 - \alpha_L^2$.

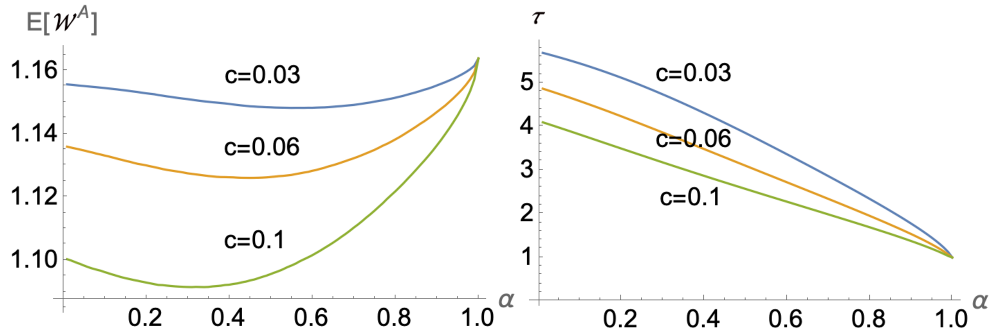

Figure 7: **More Accurate Web Search Initially Worsens the Accepted Payoff.** As Theorem 6 claims, if $u = -\infty$, the expected search reward can fall in accuracy. At left, we vary search cost c, and fix $N = 5$ options. W initially falls in α, then rises and peaks at $\alpha = 1$. At right, we present the corresponding expected search duration. Here, N is similar to the book search analysis in De Los Santos et al. (2012).

The strong search substitution effect most strongly manifests itself near accuracy $\alpha = 0$, Sam's search intensity initially falls so fast as α rises that *his search outcome worsens*. But for large α, the search outcome improves in accuracy (see Figure 7).

The proof tracks accuracy simultaneously impacts search time and outcome. Since the *marginal value of information* $\mathcal{M}(\alpha, c) \equiv \partial\mathcal{V}/\partial\alpha$ exists, it obeys $\mathcal{M}(\alpha, c) \geq 0$, by Blackwell's Theorem. By the Envelope Theorem[29] and (15), we have $\partial\mathcal{V}/\partial c = -\tau(\alpha, c)$. So:

$$\frac{\partial\tau}{\partial\alpha} = -\frac{\partial^2\mathcal{V}}{\partial\alpha\partial c} = -\mathcal{M}_c(\alpha, c) < 0. \tag{16}$$

Differentiating the optimal value formula (15) in α, and substituting (16), yields:

$$\frac{\partial E[\mathcal{W}^A]}{\partial\alpha} = \frac{\partial\mathcal{V}}{\partial\alpha} + c\frac{\partial\tau(\alpha, c)}{\partial\alpha} = \mathcal{M}(\alpha, c) - c\mathcal{M}_c(\alpha, c). \tag{17}$$

So the search outcome initially worsens since $\mathcal{M}(\alpha, c)$ is strictly convex in c near 0.[30]

Our second case posits a quit payoff $u > -\infty$, so that Sam sometimes quits searching. The mean accepted option places positive weight on the quit payoff:[31,32]

[29] The derivatives of $\mathcal{V}$ exist, as shown in Online Appendix I.

[30] The marginal value $\mathcal{M}(\alpha, c)$ vanishes as $c{\downarrow}0$ — as Sam explores all options $\tau \to N$. As this holds for all $\alpha \in (0, 1)$, we have $\partial\tau/\partial\alpha \to 0$ as $c{\downarrow}0$. Then $\mathcal{M}_c(\alpha, c) = -\partial\tau/\partial\alpha$ vanishes as $c{\downarrow}0$, by (16). But $\mathcal{M}(\alpha, c) \geq 0$ by Lemma 5. So $\mathcal{M}(\alpha, c)$ is non-negative and is zero at $c = 0$, and $\mathcal{M}_c(\alpha, c) \to 0$ as $c{\downarrow}0$, so is strictly convex in c, for c near 0. We formally show $\mathcal{M}_c(\alpha, c) \to 0$ in Appendix.

[31] Theorem 6's claim that the mean accepted option falls in α for small $c > 0$ holds for $u > -\infty$.

[32] If u rises, q rises. *Sam's search falls so much that the mean accepted option falls, even though one might accept a higher quit payoff*: $\frac{\partial}{\partial u}E[\mathcal{W}^A(\alpha, c)] < 0$ for large N (Online Appendix IV).

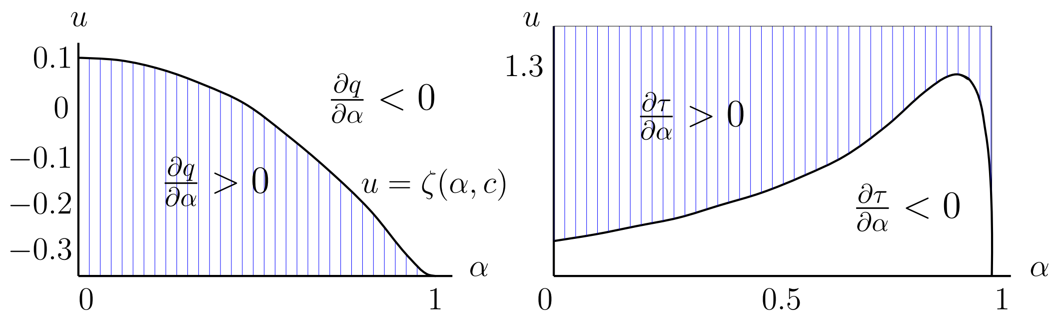

Figure 8: **Search Behavior and Accuracy** (Theorem 7). At left, the quitting chance slope in α changes sign when $u = \zeta(\alpha, c)$. At right, the expected search time falls/rises in α below/above the curve. The simulated graphs assume $c = 0.3, N = 6$.

$$E[\mathcal{W}^A(\alpha, c)] = qu + (1-q)E[\mathcal{X}^A + \mathcal{Z}^A]. \tag{18}$$

By Theorems 4–5 and Corollary 1, the CTR and search duration τ both fall if either the hidden factor dispersion falls, or the known factor dispersion rises, when $u = -\infty$. Theorem 5 implies that the CTR σ_1 and duration $\tau \equiv \Sigma_{n=1}^{N} \sigma_j$ then rise in accuracy α. But with higher quit payoffs $u > -\infty$, these two conclusions can reverse:

Theorem 7 (Rising Accuracy, II) *For low quit payoffs $u > -\infty$, the CTR and search duration τ fall in α; otherwise both rise in α. For low quit payoffs $u > -\infty$ (specifically, $u < \zeta(\alpha, c)$), the quitting chance $q(\alpha)$ rises in α; otherwise it falls in α.*

So the CTR is a completely misleading search engine accuracy measure for low quit payoffs u. And for high quit payoffs, a more accurate search engine leads Sam to spend longer searching. We also identify a conflict of interest between consumers and web shopping platforms at low u. For consumers always desire greater accuracy, but given low u, more accuracy lifts the quitting chance q, and so lowers the sale chance.

For intuition into Theorem 7, observe that more accuracy: (i) better sorts Sam's options and (ii) improves his quitting decision. For (i), assume $u = -\infty$, so that Sam never quits: The chance that he hits stage n, given known factors $\vec{x} \equiv \{x_1, x_2, \ldots, x_N\}$, is simply the chance that the n^{th} cutoff exceeds the payoffs of options 1 to $n-1$, i.e.

$$\varsigma_n(\vec{x}, \alpha) \equiv P(\alpha x_n + \zeta(\alpha, c) > \alpha x_j + \mathcal{Z}_j, \forall j < n) = \Pi_{j=1}^{n-1} H[\alpha(x_n - x_j) + \zeta(\alpha, c)|\alpha],$$

recalling (6b), with $H(\cdot|\alpha)$ the cdf of the hidden factor $\mathcal{Z} \equiv \sqrt{1-\alpha^2} Z$. Accuracy impacts $\varsigma_n(\vec{x}, \alpha)$ through two channels. First it falls in α given $x_n - x_j < 0$. Second, fixing any $\Delta \geq 0$, the hidden cdf $H[-\Delta + \zeta(\alpha, c)|\alpha]$ falls as accuracy rises (Claim 5), as

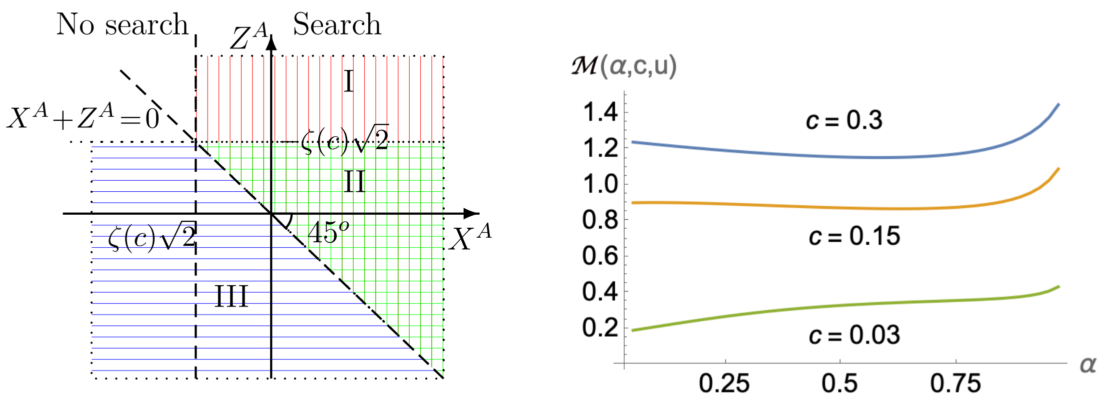

Figure 9: **Marginal Value of Web Search Information.** At left, the search decision if $N = 1, \alpha = 1/\sqrt{2}, u = 0$. Sam searches if $X^A > -\zeta(c)\sqrt{2}$ and accepts the option if $X^A + Z^A > 0$. At right, we plot the marginal value of information $\mathcal{M}(\alpha, c)$ with $N = 15$ options and quit payoff $u = -1$: $\mathcal{M}(\alpha, c)$ is increasing in α for c small.

dispersion of $\mathcal{Z}$ falls. The first channel uses Theorem 5: Sam stops sooner with larger known factor gaps. The second channel uses Theorem 4: less hidden factor dispersion reduces search duration. Since Sam does not quit before exploring the n^{th} option iff $X_n > \ell(\alpha, u, c)$, the nth search survival chance is $\sigma_n = E[\varsigma_n(\vec{X}, \alpha)\mathbb{1}_{\{X_n > \ell(\alpha,u,c)\}}]$. Since $\ell_\alpha(\alpha, u, c) > 0$ iff u is small,[33] all survival chances σ_n fall, as do the CTR and τ.

6.2 Convexity in Accuracy and Informational Nonconcavity

Arguably, with countless Google searches every hour, information acquired from search engines is quite valuable. We thus revisit the classic nonconcavity of the value of information, and find *a fortiori* strict convexity $\mathcal{V}(\alpha, c)$ in search engine accuracy. This endogenous increasing returns in search engines suggests a natural monopoly.

Let $\mathcal{A}$ be the event that Sam eventually accepts an inside option $(\mathcal{X}^A, \mathcal{Z}^A) = (\alpha X^A, \sqrt{1-\alpha^2} Z^A)$, and $\mathcal{L} \subset \mathcal{A}$ the event that Sam is *pleasantly surprised* by the hidden factor: $\mathcal{Z}^A \geq \zeta(\alpha, c)$. So while Sam would still accept a lesser prize, he would have been willing to search had he foreseen its hidden factor: $\mathcal{X}^A + \mathcal{Z}^A \geq \mathcal{X}^A + \zeta(\alpha, c)$, the cutoff for exploring $(\mathcal{X}^A, \mathcal{Z}^A)$, by Lemma 2. Also, $(\mathcal{X}^A, \mathcal{Z}^A)$ dominates Sam's fall back options, as well as the lower cutoffs of future options. Sam accepts $(\mathcal{X}^A, \mathcal{Z}^A)$.

Lemma 5 *The marginal value of information is* $\mathcal{M}(\alpha, c) = P(\mathcal{L})E[X^A|\mathcal{L}] \geq 0$.

[33]Observe from (14) that $\ell_\alpha(\alpha, u, c) > 0$ iff the outside option u is below some real $\bar{u}$. For $\ell_\alpha(\alpha, u, c) = -[\ell(\alpha, u, c) + \zeta_\alpha(\alpha, c)]/\alpha \gtrless 0$ as $u \lessgtr \bar{u}$, since $\ell(\alpha, u, c)$ increases in u.

Intuitively, $\mathcal{L}$ is the event that an option impacts Sam's search, and lifts his payoff. Sam's value (15) is $\mathcal{V}(\alpha, c) = u[1 - P(\mathcal{A})] + E[(\alpha X^A + \sqrt{1-\alpha^2} Z^A)\mathbb{1}_{\mathcal{A}}] - \tau c$. By the Envelope Theorem, the derivative in α is simply the partial derivative:

$$\mathcal{M}(\alpha, c) = \frac{\partial \mathcal{V}}{\partial \alpha} = E[(X^A - \frac{\alpha}{\sqrt{1-\alpha^2}} Z^A)\mathbb{1}_{\mathcal{A}}]. \tag{19}$$

For intuition into Lemma 5, assume $N = 1$, $u = 0$, and accuracy $\alpha = \sqrt{1-\alpha^2} = 1/\sqrt{2}$. Consult the left panel of Figure 9. As Sam searches if $X^A > \zeta(1/\sqrt{2}, c)\sqrt{2}$ by (14), Sam accepts in event $\mathcal{A} = \{X^A > \zeta(1/\sqrt{2}, c)\sqrt{2}, X^A + Z^A > 0\}$, or area $I + II$:

$$\frac{\partial \mathcal{V}}{\partial \alpha} = E[(X^A - Z^A)\mathbb{1}_{I+II}] = E[X^A \mathbb{1}_{II} - Z^A \mathbb{1}_{I+II}] + E[X^A \mathbb{1}_I]$$

The first bracketed term vanishes: Reflecting area $I + II$ in the dashed diagonal line into area III preserves the $N(0, 1)$ probability densities of X^A and Z^A. So the integral of Z^A in I + II is the integral of $-X^A$ in III. Since X^A and Z^A are independent:

$$E[X^A \mathbb{1}_{II} - Z^A \mathbb{1}_{I+II}] = E[X^A \mathbb{1}_{II} + X^A \mathbb{1}_{III}] = E[X^A \mathbb{1}_{\{Z^A < -\zeta(c)\sqrt{2}\}}] = 0.$$

Since $\mathcal{L} = \mathcal{A} \cap \{Z^A \geq -\zeta(c)\sqrt{2}\}$ is area I, $E[X^A \mathbb{1}_I] = E[X^A \mathbb{1}_{\mathcal{L}}] = P(\mathcal{L})E[X^A|\mathcal{L}]$. □

The marginal value of information is higher at $\alpha = 1$ than at 0 (Figure 9, right).

Theorem 8 *The marginal value of information is higher near a perfect search engine than a bad one: $\mathcal{M}(0, c) \leq \mathcal{M}(1, c)$, with equality iff $N = 1$ and $u = -\infty$. Also, this marginal value rises ($\mathcal{M}_\alpha(\alpha, c) > 0$) for small search costs $c > 0$, and $\alpha \in (0, 1)$.*

To see that $\mathcal{M}(\alpha, c)$ can rise in accuracy, consider the extreme case $\alpha = 0$. When $u > \zeta(0, c)$, Sam will not search, and more accuracy has no value, i.e. $\mathcal{M}(0, c) = 0$.

Next, assume $\alpha > 0$. For small costs $c > 0$, Sam explores all options and accepts the best if its payoff exceeds u. Sam can only explore fewer options as α rises, and so the accepted option's known factor $E[X^A|\mathcal{L}]$ rises in α. Also, the acceptance decision does not depend on α. But $P(\mathcal{L}) \equiv P(\mathcal{A} \cup \{Z^A \geq \zeta(\alpha, c)/\sqrt{1-\alpha^2}\})$ rises in α as $\zeta(\alpha, c)/\sqrt{1-\alpha^2}$ falls in α (see Claim 9). Both factors lift $\mathcal{M}(\alpha, c)$, by Lemma 5.

Corollary 2 (Search Engine Synergy) *Accuracy α and* usability *$-c$ are complements to the web size N, namely $\partial^2 \mathcal{V}/\partial N \partial \alpha \geq 0$ and $\partial^2 \mathcal{V}/\partial N \partial c \leq 0$.*

So as the web size N has exploded, search engine accuracy has risen and costs fallen.

7 How Does Search Change with More Options?

We now consider what happens when the number of options N grows, as has occurred. Stopping decisions depend on the order statistic gaps $\mathcal{X}_n^N - \mathcal{X}_{n+1}^N$. We next use dispersion logic to show that with more total options N, search lasts longer since these gaps stochastically shrink, thereby depressing quitting, striking, and recall.

Let $\mathcal{K}_n$ be the chance of *striking* the stage-n option, where $\mathcal{E}_n = \mathcal{K}_n + \mathcal{R}_n$.

Theorem 9 (More Options) *The quitting, striking and recall chances $\mathcal{Q}_n^N$, $\mathcal{K}_n^N$, and $\mathcal{R}_n^N$ in any stage n all weakly fall in the number of options N. Search duration rises with additional options, and the recall moment happens later.*

Sam's welfare rises in the number of options N, since the best are first presented. But more subtly, search duration optimally rises in N. For instance, many firms receive far more web applications for every position than in pre-web days.[34] With a larger applicant pool, the options presented first are better, but employers grow more ambitious. Does vacancy duration rise? For the same reason, those searching for mates online expect to remain unmatched much longer, given the wealth of options.

For insight into Theorem 9, assume we draw $N+1$ options rather than N. Given any realization of the smallest known factor $\mathcal{X}_{N+1} = \chi_{N+1}$, by the Markov property of order statistics, the joint distribution of known factors $\{\mathcal{X}_1, ..., \mathcal{X}_N\}$ is the same as N i.i.d. draws from the left-truncated distribution with cdf $G(\chi)/[1 - G(\chi_{N+1})]$. This is less dispersed than the original distribution G, and so the order statistics gaps are smaller — akin to the proof of Theorem 5. So search continues more often at every subsequent stage than when there are only N options (see footnote 14). This intuition highlights the link between log-concavity and dispersion for search theory.

Ours is a nonstationary search model with a finite number of options, and ranked order statistics of high known factors. Does the standard stationary search model reasonably predict Sam's behavior in the limit with $N \uparrow \infty$ options? Sam never quits in this limit — since the outside option is dominated by *some* inside option. But recall

[34]Using CPS data, Kuhn and Skuterud (2004) find that web job search does not raise the job-finding rate. Martellini and Menzio (2020) likewise explain the stability of the unemployment rate despite information technology improvements: A more efficient matching function might not reduce the unemployment rate in the Diamond-Mortensen-Pissarides model. We model the nonstationary search process, well-founding their matching process. Firms get more applications means N rises.

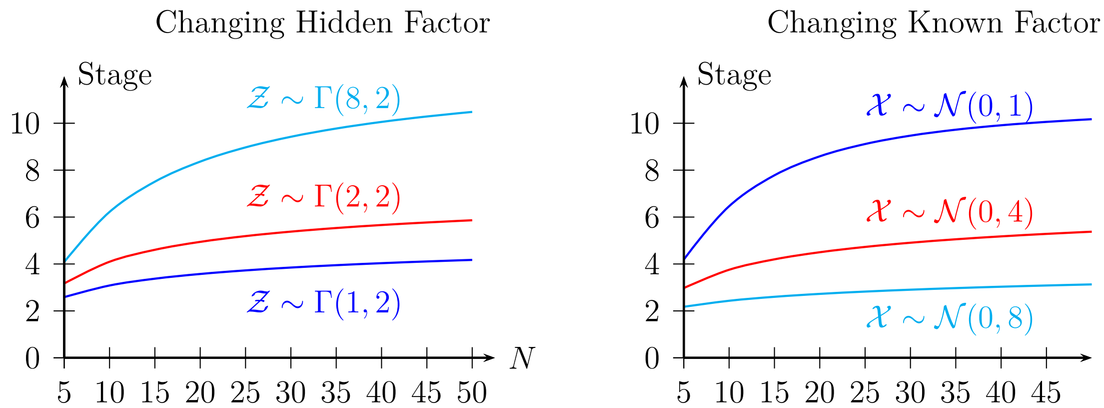

Figure 10: **Dispersion and Search Duration.** Assume $c = 0.2$ and $u = -\infty$. Left: Duration rises in the hidden factor dispersion for $\mathcal{X} \sim \mathcal{N}(0, 2.8)$ (Theorem 4). Right: Duration falls in the known factor $\mathcal{X}$ dispersion for $\mathcal{Z} \sim \Gamma(1.2, 2)$ (Theorem 5).

is forever a feature, and its probability need not vanish if, say, known factors $\mathcal{X}$ are exponentially distributed. For then order statistic gaps are constant (footnote 14).

The striking and recall hazard rates converge to their stationary limits only if all top order statistic gaps $\mathcal{X}_n^N - \mathcal{X}_{n+1}^N$ vanish as $N \uparrow \infty$. This happens[35] if the distribution G has a *thin (right) tail*, namely, if $\lim_{\chi \uparrow G^{-1}(1)} g(\chi)/[1-G(\chi)] = \infty$. This excludes our knife-edge exponential case, given the constant hazard rate $n\lambda > 0$.

Theorem 10 (Many Options) *Fix stage n. Let $N \to \infty$. Then $\mathcal{Q}_n^N \to 0$. If G has a thin tail, $\mathcal{R}_n^N \to 0$ and $\mathcal{K}_n^N \to 1 - H(\zeta(c))$. If not, $\mathcal{R}_n^N \to \mathcal{R}_n^\infty > 0$ and $\mathcal{K}_n^N \to \mathcal{K}_n^\infty > 1 - H(\zeta(c))$, where $\mathcal{R}_n^\infty + \mathcal{K}_n^\infty < 1$. The limit recall chance $\mathcal{R}_n^\infty$ rises in $\mathcal{X}$ dispersion.*[36]

When the known factor lacks a thin tail, optimal behavior with a vast number of options differs much from the infinite horizon search model.[37] Sam recalls with a boundedly positive chance. Sam also strikes more often than justified by the hidden noise. The reason is that the gaps between consecutive known factors don't vanish; this gives an extra incentive to strike now — as next period is worse than this one.[38]

[35]As the hazard rate $g/[1-G]$ is non-decreasing, when $\lim_{\chi \uparrow G^{-1}(1)} g(\chi)/[1-G(\chi)]$ exists and is positive. A thin tail means $\lim_{\chi \uparrow G^{-1}(1)} g(\chi)/[1-G(\chi)] < \infty$. Table 1 lists thin tail distributions.

[36]This also result holds at the interim stage. In here we show that, conditioned on realized known factors, the stage-n conditional recall chance rises as the gaps, $\chi_i - \chi_{i+1}$, for $i \leq n$, weakly increase.

[37]For example the difference in search duration (Corollary 1) can be substantial — in here we show that the ratio between the search duration in an infinite horizon stationary search model and that in our model with a vast number of options can explode to infinity as $c \downarrow 0$.

[38]The two models also make different predictions over the expected search cost τc. We show here that when the known factor lacks a thin tail, the expected search cost with large N always vanishes as $c \to 0$. But in an infinite horizon search model τc vanishes if and only if H also has a thin tail.

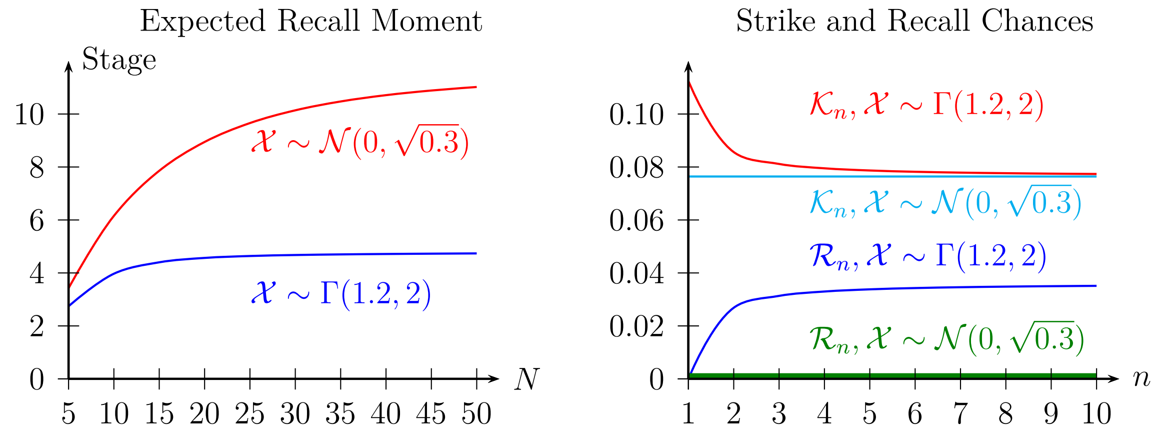

Figure 11: **More Options Lift Duration and Search Intensifies.** Left: The mean recall moment when search ends (Lemma 3) rises in N, by Theorem 9. Put $\mathcal{X} \sim \mathcal{N}(0, \sqrt{0.3})$ and $\mathcal{X} \sim \Gamma(1.2, 2)$ with variance 0.3. Right: By Theorem 10, as $N \uparrow \infty$, the recall chance $\mathcal{R}_n$ vanishes if $\mathcal{X}$ has a fat tail (Gaussian), but otherwise is strictly positive (here, Gamma). In both panels, $\mathcal{Z} \sim \Gamma(2.8, 2)$, $c = 0.2$, and $u = -\infty$.

8 Conclusion

We develop and characterize a tractable twist on the benchmark stationary search model to capture economic settings with prior information or web search. We assume finitely many options whose payoffs are the sum of known and hidden factors with logconcave densities. This generates random families of Weitzman search models.

With logconcave distributions, search intensifies over time: quitting, recall and exercise chances increase. If recall occurs, older options are recalled more often.

We have resolved a basic but long outstanding open problem — dispersion is the distribution shifts increasing search duration. In our two factor model, more hidden factor dispersion prolongs search, and more known factor dispersion truncates search.

Improved web search engines always help Sam: A more accurate search engine always reduces Sam's search duration. But initially as accuracy rises, Sam relaxes his search effort so quickly that his expected search outcome worsens. We also find a strong manifestation of the nonconcavity of informational value — namely, it persists at all accuracy levels, and suggests that Google might be a natural monopoly.

Finally, Sam searches more with more total options. In the limit of an exploding number of options betrays a failure of lower hemicontinuity: Stationary search poorly predicts finite real world behavior unless the known factor has a thin tail.

Our two factor model has practical economic applications. For example, online matching has improved over time (Regnerus, 2017). If new technology raises signal precision, known factors grow more disperse, and search duration falls (Theorem 5). But if it instead more cheaply yields the same quality signal, then people grow pickier.

Our paper opens the door to formal analyses of age-old behavioral topics on the curse of choice (Chernev et al., 2015) — like Toffler's 1970 concept of "overchoice". Indeed, with more options, Sam's investigative look-see efforts rise. Our model also captures decision fatigue — smart behavioral model of an unmodeled ex ante stage in which Sam rank orders known factors is an intriguing open problem.

Finally, De Los Santos et al. (2012) study an online book market and find that finding price discounts does not raise the stopping chance of the searchers. They conclude that consumers do not search sequentially in web search. We argue here in a calibrated model that if the price discounts belong to the known (and not hidden) factors, then they can induce consumers to search *longer*, even if search is sequential.

A Optimal Stopping: Proof of Lemma 4

All claims hold at stage N: For it reduces to a one-shot search problem with a fallback option: $V_N(\Omega)=\Omega$ for $\Omega<\bar{w}_N$ as the best option Ω_N is exercised. Then $V_N'(\Omega)=1$.

Assume all claims at stage $n+1$. Search stops at stage n if $\Omega_n \geq \bar{w}_{n+1}$. By (4), $V_n(\Omega)=\Omega$ on $[\bar{w}_{n+1},\infty)$ and so $V_n'(\Omega)=1$, i.e., the stopping chance. Sam searches at stage $n+1$ if $\Omega_n<\bar{w}_{n+1}$. Then $V_n'(\Omega_n)=F_{n+1}(\Omega_n)V_{n+1}'(\Omega_n)$ by (4). Since V_{n+1}' jumps up at $\bar{w}_N<\cdots<\bar{w}_{n+2}$, so does V_n'. Now, $1=V_n'(\bar{w}_{n+1}+)>V_n'(\bar{w}_{n+1}-)=F_{n+1}(\bar{w}_{n+1})V_{n+1}'(\bar{w}_{n+1}-)$ as $V_{n+1}'(\bar{w}_{n+1}-)<1$ by assumption, and $F_{n+1}(\bar{w}_{n+1})<1$. Then V_n' exists except at jumps $\bar{w}_N<\cdots<\bar{w}_{n+1}$. If $\Omega_n<\bar{w}_{n+1}$, then Sam enters stage $n+1$ and recalls Ω_n with chance $V_n'(\Omega_n)=F_{n+1}(\Omega_n)V_{n+1}'(\Omega_n)$. As F_{n+1} has full support and V_{n+1} is convex, $F_{n+1}V_{n+1}'$ rises, and $F_{n+1}(\Omega_n)<1$. So V_n is strictly convex and $V_n'(\Omega)<1$ for all $\Omega<\bar{w}_{n+1}$. The last claim holds since Sam enters stage k with best-so-far w iff $w=\Omega_{k-1}<\bar{w}_k$, and he recalls w iff $w=\Omega_k\geq\bar{w}_{k+1}$, by Lemma 1.

Finally, $V_n(\Omega)$ *grows weakly steeper for all* $n=1,\ldots,N$. The claim holds if $n=N$, as $V_N'(\Omega)=1$. For $n<N$, $V_n'(\Omega)=1$ for $\Omega\geq\bar{w}_{n+1}$ and $V_n'(\Omega)=F_{n+1}(\Omega)V_{n+1}'(\Omega)<1$ otherwise. As $\bar{w}_{n+1}$ falls in c by (4), and $V_{n+1}'(\Omega)$ weakly rises in c, so does $V_n'(\Omega)$. □

B Search Over Time: Selection Effect Proofs

B.1 Stochastic Shifts of the Known Factor

The *ex ante* probability density when we hit stage n with known factor $\mathcal{X}_n = \chi$ is:

$$\eta(\chi, c, n, N) \equiv \delta(\chi, c)^{n-1} G(\chi)^{N-n} g(\chi). \tag{20}$$

By (7) and (20), the *stage-n conditional expectation* operator $E_{\mathcal{X}_n}$ is given by the cdf

$$P(\mathcal{X}_n \leq a|\text{stage } n) = \frac{N\binom{N-1}{n-1}\int_{u-\zeta(c)}^{a} \eta(\chi, c, n, N)d\chi}{\sigma_n} = \frac{\int_{u-\zeta(c)}^{a} \eta(\chi, c, n, N)d\chi}{\int_{u-\zeta(c)}^{\infty} \eta(\chi, c, n, N)d\chi} \tag{21}$$

Claim 1 (More Options) *If Sam hits stage n, the known factor $\mathcal{X}_n$ stochastically rises in the number N of options, search cost $c > 0$, and quit payoff u, and falls in n.*

Proof: We argue that the cdf (21) falls in N, c and u, and rises in n. For by (20):

$$\frac{\partial}{\partial a} \log \left[\int_{u-\zeta(c)}^{a} \eta(\chi, c, n, N)d\chi\right] = \frac{\delta(a, c)^{n-1} G(a)^{N-n} g(a)}{\int_{u-\zeta(c)}^{a} \delta(\chi, c)^{n-1} G(\chi)^{N-n} g(\chi)d\chi}. \tag{22}$$

We repeatedly use logsupermodularity. Since $G(a)/G(\chi) > 1$ if $\chi < a$, the RHS of (22) rises in N. So the bracketed term in (22) is LSPM in (N, a), and ratio (21) falls in N, as $a < \infty$. Since $\delta(\chi, c)$ is LSPM by Claim 2 below, $\delta(a, c)/\delta(\chi, c)$ rises in c, if $\chi < a$. As $u - \zeta(c)$ rises in c, the RHS of (22) rises in c and u. So $\int_{u-\zeta(c)}^{a} \eta(\chi, c, n, N)d\chi$ is LSPM in (c, a) and (u, a). So the ratio (21) falls in c and u. Finally, $\delta(\chi, c)/G(\chi)$ falls in χ, by Claim 2, i.e. $\delta(\chi, c)/G(\chi) > \delta(a, c)/G(a)$ for $\chi < a$. So (22) falls in n, and the bracketed term in (22) is LSPM in (n, a). So the ratio (21) rises in n. □

Claim 2 (Delay Chance) *$\delta(\chi, c)$ falls in c and is LSPM; also, $\delta(\chi, c)/G(\chi)$ falls in χ.*

Proof: Put $s = a - \chi$ in (5). Then $\delta(\chi, c) = \int_0^\infty H(\zeta(c) - s)\, g(s + \chi)ds$. Then $\zeta'(c) < 0$ implies $\delta_c(\chi, c) < 0$. Since $H(\zeta(c) - s)$ and $g(s + \chi)$ are LSPM in $(\zeta(c), s)$ and $(s, -\chi)$, resp., and partial integration preserves LSPM (Karlin and Rinott, 1980), $\delta(\chi, c)$ is LSPM in $(\zeta(c), -\chi)$, and so in (c, χ). Integrating (5) by parts, the delay chance is

$$\delta(\chi, c) = -H(\zeta(c))\, G(\chi) + \int_0^\infty h(\zeta(c) - s)\, G(s + \chi)ds. \tag{23}$$

Since $G(\chi)$ is log-concave, $G(s+\chi)/G(\chi)$ falls in χ, and thus so does $\delta(\chi, c)/G(\chi)$. □

B.2 Conditional Stopping Chances: Proof of Theorem 1

By prospective independence, Sam explores $n \leq N$ options and then quits with chance:

$$q_n = \binom{N}{n}\delta(u-\zeta(c),c)^n G(u-\zeta(c))^{N-n}. \tag{24}$$

Then the quitting hazard rate $\mathcal{Q}_n \equiv q_n/\sigma_n$ rises in the stage n if σ_n falls and q_n rises in n. The survival chance σ_n that search lasts at least n stages must fall in n. Next:

Claim 3 (Quitting Chance) *The quitting chance q_n rises in n for all small costs $c > 0$, is hump-shaped in n for intermediate c, and falls in n for all large c.*

Proof: By (24), the ratio $q_{n+1}/q_n = [(N-n)/(n+1)]\delta(u-\zeta(c),c)/G(u-\zeta(c))$ falls in $n = 0, 1 \ldots, N-1$. For if $\delta/G < 1/N$, then $q_{n+1}/q_n < 1$ for all $n = 0, \ldots, N-1$, and so q_n falls in n. If $\delta/G > N$, then $q_{n+1}/q_n > 1$ for all $n = 0, \ldots, N-1$, and so q_n rises in n. Finally, if $1/N < \delta/G < N$, then q_n is hump-shaped in n from 0 to N.

Next, we show that $\delta(u-\zeta(c),c)/G(u-\zeta(c))$ falls from ∞ to 0 as for $c \in [0,\infty)$. For (5) implies:

$$\frac{\delta(u-\zeta(c),c)}{G(u-\zeta(c))} = \int_{-\zeta(c)}^{\infty} H(-s)\frac{g(s+u)}{G(u-\zeta(c))}ds. \tag{25}$$

Since $\zeta'(c) < 0$, (25) falls in c, vanishing as $c \to \infty$ (for then $\zeta(c) \to -\infty$ by (3)), exploding as $c \to 0$ (for then $\zeta(c) \to \infty$, and thus $H(\zeta(c) - r) \to 1$). So, (25) implies:

$$\lim_{c\to 0}\frac{\delta(u-\zeta(c),c)}{G(u-\zeta(c))} = \lim_{c\to 0}\frac{\int_0^\infty H(\zeta(c)-r)g(r+u-\zeta(c))dr}{G(u-\zeta(c))} = \lim_{\zeta(c)\to\infty}\frac{[1-G(u-\zeta(c))]}{G(u-\zeta(c))}$$

i.e. an infinite limit. So $\exists \bar{c} > \underline{c}$ s.t. (1) $\delta(u-\zeta(c),c)/G(u-\zeta(c)) > N$ if $c < \underline{c}$, and so q_n rises in n; (2) $\delta(u-\zeta(c),c)/G(u-\zeta(c)) \in [1/N, N]$ if $c \in [\underline{c}, \bar{c}]$, and q_n is hump-shaped in n; and (3) $\delta(u-\zeta(c),c)/G(u-\zeta(c)) < 1/N$ if $c > \bar{c}$, and so q_n falls in n. □

Consider the temporary perspective of a partly omniscient observer, knowing one realized option (χ, z). Let $w^*(\chi, z) \equiv \min(\chi + z, \bar{w})$, for the reservation prize $\bar{w} = \chi + \zeta(c)$. Modifying the n-survival event (6a)–(6c), Sam exercises (χ, z) at stage n iff

$$w^*(\chi, z) > u \tag{26a}$$

$$\mathcal{X}' + \zeta(c) > w^*(\chi, z) > \mathcal{X}' + \mathcal{Z}' \text{ for } n-1 \text{ options } (\mathcal{X}', \mathcal{Z}') \tag{26b}$$

$$w^*(\chi, z) > \mathcal{X}'' + \zeta(c) \text{ for } N-n \text{ options } (\mathcal{X}'', \mathcal{Z}''). \tag{26c}$$

We claim (26) implies n-survival. Then (6) holds if $w^*(\chi, z) = \bar{w}$. And if $w^*(\chi, z) = \chi + z < \bar{w}$, all $\mathcal{X}' + \zeta(c)$ and $\bar{w}$ exceed $\chi + z$, all $\mathcal{X}' + \mathcal{Z}'$, all $\mathcal{X}'' + \zeta(c)$, and u; so Sam explores (χ, z) and all $(\mathcal{X}', \mathcal{Z}')$. We claim Sam exercises (χ, z) in event (26).

Claim 4 (Exercise) *Sam strikes* (χ, z) *given* (26) *and* $\mathcal{X}' > \chi$ *for all* $n-1$ *options* $(\mathcal{X}', \mathcal{Z}')$. *Sam recalls* (χ, z) *if* (26) *and* $\mathcal{X}' \leq \chi$ *for some of the* $n-1$ *options* $(\mathcal{X}', \mathcal{Z}')$.

Proof: As $w^* \equiv w^*(\chi, z) > u$, Sam never quits in a one option world, and so never quits with N options. For which inside option does Sam exercise, and how?

Assume (26). First, (χ, z) *blocks* (*ex ante* dominates) the $N-n$ options with $w^* > \mathcal{X}'' + \zeta(c)$ — Sam never later explores them. We next show that Sam explores all $n-1$ options $(\mathcal{X}', \mathcal{Z}')$ in some order, and then exercises (χ, z). First, Sam explores (χ, z) at some stage, as no option $(\mathcal{X}', \mathcal{Z}')$ blocks it: For $\bar{w} > \mathcal{X}' + \mathcal{Z}'$, by (26).

Next, as $\mathcal{X}' + \zeta(c) > w^*$, either $\mathcal{X}' + \zeta(c) > \bar{w}$ and so Sam explores $(\mathcal{X}', \mathcal{Z}')$ before (χ, z), or $\bar{w} \geq \mathcal{X}' + \zeta(c) > \chi + z$, and so (χ, z) delays Sam. In either case, (χ, z) does not block $(\mathcal{X}', \mathcal{Z}')$. Finally, as any two of the $n-1$ options $(\mathcal{X}'_A, \mathcal{Z}'_A)$ and $(\mathcal{X}'_B, \mathcal{Z}'_B)$ obey $\mathcal{X}'_A + \zeta(c) > w^* > \mathcal{X}'_B + \mathcal{Z}'_B$ by (26), no option $(\mathcal{X}'_B, \mathcal{Z}'_B)$ blocks another $(\mathcal{X}'_A, \mathcal{Z}'_A)$. So Sam eventually explores all $(\mathcal{X}', \mathcal{Z}')$, exercising (χ, z) at stage n, as $\chi + z > \mathcal{X}' + \mathcal{Z}'$.

When $\mathcal{X}' > \chi$ for $n-1$ known factors, the option (χ, z) is the n^{th} option, and so Sam strikes (χ, z). The last claim follows at once from (26) and the first claim. □

We first introduce the interim random variable $\mathcal{W}^* \equiv \mathcal{X} + \min(\mathcal{Z}, \zeta(c)) \equiv w^*(\mathcal{X}, \mathcal{Z})$. The interim n-exercise event (26) is the n-survival event (6) if $w^* = \chi + \zeta(c)$, and has chance

$$\Lambda_n(w) \equiv N \binom{N-1}{n-1} \delta(w - \zeta(c), c)^{n-1} G(w - \zeta(c))^{N-n} \tag{27}$$

for $w > u$, and $\Lambda_n(w) = 0$ otherwise. Next, convoluting densities for $z = \zeta(c) - s$ and $\chi = w^* - \min(z, \zeta(c)) = w^* - \zeta(c) + \max(s, 0)$, we see that $\mathcal{W}^*$ has *ex ante* probability density

$$\phi(w^*) \equiv \textstyle\int_{-\infty}^{\infty} h(\zeta(c) - s) g(w^* - \zeta(c) + \max(s, 0)) ds. \tag{28}$$

So, the n-survival chance (7) as $\sigma_n = E_g[\Lambda_n(\mathcal{X} + \zeta(c))]$, and the n-exercise chance as

$$e_n = E_\phi[\Lambda_n(\mathcal{W}^*)] = N \binom{N-1}{n-1} \int_u^\infty \delta(w^* - \zeta(c), c)^{n-1} G(w^* - \zeta(c))^{N-n} \phi(w^*) dw^* \tag{29}$$

Steps 1–3 use the operator $E_{\mathcal{X}_n}$ to compute the conditional chances $\mathcal{E}_n, \mathcal{K}_n$, $\mathcal{R}_n$.

Step 1 (Exercise Chance Formula) *The conditional exercise chance* $\mathcal{E}_n$ *rises in* n*, and*

$$\mathcal{E}_n = 1 - H(\zeta(c)) + E_{\mathcal{X}_n}\left[\int_0^\infty h(\zeta(c)-s)\frac{g(s+\mathcal{X}_n)}{g(\mathcal{X}_n)}ds\right]. \tag{30}$$

The $1-H(\zeta(c))$ term integrates the top rectangle of the striking set in Figure 1: Sam always strikes if $z_n > \zeta(c)$ — for if he enters stage n, then $\chi_n + \zeta(c) > \Omega_{n-1}$ by (6a) and (6b) — and the striking event $w_n \geq \max\{\bar{w}_{n+1}, \Omega_{n-1}\}$ holds (see Lemma 1). We prove that the second term in (30) is the conditional exercise chance if $z_n \leq \zeta(c)$.

Proof of Step 1: By (7) and (29), rewrite the conditional chance $\mathcal{E}_n \equiv e_n/\sigma_n$ as

$$\mathcal{E}_n = \frac{\int_u^\infty \delta(w^*-\zeta(c),c)^{n-1}G(w^*-\zeta(c))^{N-n}\phi(w^*)dw^*}{\int_{u-\zeta(c)}^\infty \delta(\chi,c)^{n-1}G(\chi)^{N-n}g(\chi)d\chi} = \frac{\int_{u-\zeta(c)}^\infty \frac{\phi(\chi+\zeta(c))}{g(\chi)}\eta(\chi)d\chi}{\int_{u-\zeta(c)}^\infty \eta(\chi)d\chi} \tag{31}$$

writing $\chi = w^* - \zeta(c)$, where $\eta(\chi) = \eta(\chi, c, n, N) \equiv \delta(\chi, c)^{n-1}G(\chi)^{N-n}g(\chi)$ by (20).

Since $\eta(\chi)$ is the density in (21) of $\mathcal{X}_n$ in the operator $E_{\mathcal{X}_n}$, (28) yields (30), as

$$\mathcal{E}_n = E_{\mathcal{X}_n}\left[\frac{\phi(\mathcal{X}_n+\zeta(c))}{g(\mathcal{X}_n)}\right] = E_{\mathcal{X}_n}\left[\frac{\int_{-\infty}^\infty h(\zeta(c)-s)g(\mathcal{X}_n + \max(s,0))ds}{g(\mathcal{X}_n)}\right].$$

Then (30) rises in n, as $\mathcal{X}_n$ stochastically falls in n (Claim 1), and $\frac{g(s+x)}{g(x)}$ falls in x.□

Step 2 (Striking Chance Formula) *The conditional striking chance* $\mathcal{K}_n$ *equals*

$$\mathcal{K}_n = 1 - H(\zeta(c)) + E_{\mathcal{X}_n}\left(\int_0^\infty h(\zeta(c)-s)\frac{g(s+\mathcal{X}_n)}{g(\mathcal{X}_n)}\left[\frac{\int_s^\infty H(\zeta(c)-t)g(t+\mathcal{X}_n)dt}{\int_0^\infty H(\zeta(c)-t)g(t+\mathcal{X}_n)dt}\right]^{n-1}ds\right).$$

Proof: By Claim 4, Sam strikes if (26) holds and $\mathcal{X}' > \chi$ for $n-1$ options $(\mathcal{X}', \mathcal{Z}')$. The interim density $\phi_I(w^*)$ below modifies the *ex ante* density $\phi(w^*)$ in (28), conditioning on $\mathcal{X}' > \chi = w^* - \zeta(c) + \max\{0, s\}$ for $n-1$ options $(\mathcal{X}', \mathcal{Z}')$ — and so we divide by $P(\mathcal{X}' + \zeta(c) > w^* > \mathcal{X}' + \mathcal{Z}')$ for each of these $n-1$ options:

$$\phi_I(w^*) = \int_{-\infty}^\infty h(\zeta(c)-s)g(w^*-\zeta(c)+\max\{s,0\})\left[\frac{\int_{\max\{0,s\}}^\infty H(\zeta(c)-t)g(t+w^*-\zeta(c))dt}{\int_0^\infty H(\zeta(c)-t)g(t+w^*-\zeta(c))dt}\right]^{n-1}ds.$$

Sam strikes at stage n with *ex ante* chance $k_n = E_{\phi_I}[\Lambda_n(\mathcal{W}^*)] = \int_u^\infty \Lambda_n(w^*)\phi_I(w^*)dw^*$, recalling (27). By the logic for (30), as seen in (31), the conditional striking chance is

$$\mathcal{K}_n \equiv \frac{k_n}{\sigma_n} = \frac{\int_{u-\zeta(c)}^{\infty}[\phi_I(\chi+\zeta(c))/g(\chi)]\eta(\chi)d\chi}{\int_{u-\zeta(c)}^{\infty}\eta(\chi)d\chi} = E_{\mathcal{X}_n}\left[\frac{\phi_I(\mathcal{X}_n+\zeta(c))}{g(\mathcal{X}_n)}\right].$$

We can rewrite this expression as the desired formula by the proof logic in Step 1. □

Step 3 (Recall Chance Formula) *The recall chance is* $\mathcal{R}_n = E_{\mathcal{X}_n}[B(\mathcal{X}_n, n)]$, *where*

$$B(\chi,n) \equiv \int_0^{\infty} h\left(\zeta(c)-s\right)\frac{g(s+\chi)}{g(\chi)}\left(1-\left[\frac{\int_s^{\infty}H(\zeta(c)-t)g(t+\chi)dt}{\int_0^{\infty}H(\zeta(c)-t)g(t+\chi)dt}\right]^{n-1}\right)ds. \quad (32)$$

Proof: Formula (32) follows at once from Steps 1 and 2 and $\mathcal{R}_n = \mathcal{E}_n - \mathcal{K}_n$. □

Lastly, we prove that $\mathcal{R}_n$ increases in n. This is subtle, for while $B(\chi, n)$ in (32) increases in n, the parenthetical factor in (32) rises in χ, and $\mathcal{X}_n$ falls stochastically in n. But Step 4 implies that $B(\chi, n)$ falls in χ, and so $\mathcal{R}_n = E_{\mathcal{X}_n}[B(\mathcal{X}_n, n)]$ rises in n.

Step 4 (Recall Chance and Stage) $B(\chi, n)$ *weakly falls in* χ, *and* $\mathcal{R}_n$ *rises in* n.

Proof: As Sam strikes (χ, z) if $z \geq \zeta(c)$, if he recalls (χ, z) then he once passed it over, and so $z < \zeta(c)$; thus, $w^*(\chi, z) = \chi + z < \chi + \zeta(c) = \bar{w}$. Also if Sam recalls (χ, z) at stage n, then he must have explored (χ, z), and so $\chi > \mathcal{X}_n$. Given $w^*(\chi, z) = w \equiv \chi + z$ and $\chi > \mathcal{X}_n$, event (26b) is equivalent to the intersection of the next two events:

$$\mathcal{X}_n + \zeta(c) > w > \mathcal{X}_n + \mathcal{Z}_n \text{ and } \chi > \mathcal{X}_n, \quad (30')$$

$$\mathcal{X}' > \mathcal{X}_n \text{ and } w > \mathcal{X}' + \mathcal{Z}' \text{ for } n-2 \text{ prior options } (\mathcal{X}', \mathcal{Z}'). \quad (30'')$$

By (30′), $(\mathcal{X}_n, \mathcal{Z}_n)$ obeys both inequalities in (26b) and (χ, z) ranks before $(\mathcal{X}_n, \mathcal{Z}_n)$. By (30″), $n-2$ other options satisfy (26b), also ranking before $(\mathcal{X}_n, \mathcal{Z}_n)$. By Claim 4, the *ex ante* chance of recall r_n is the *ex ante* chance of (30′), (30″), (26a) and (26c).

To compute r_n, let $\Upsilon(w)$ be the density of (i) the interim variable $\mathcal{W} \equiv \mathcal{X} + \mathcal{Z} = w$ for the target option, (ii) a given option $(\mathcal{X}_n, \mathcal{Z}_n)$ obeying (30′) and (iii) $n-2$ options $(\mathcal{X}', \mathcal{Z}')$ obeying (30″). By Claim 4, (26a), and (26c), the stage-n recall chance is

$$r_n = N(N-1)\binom{N-2}{N-n}\int_u^{\infty}\Upsilon(w)G(w-\zeta(c))^{N-n}dw \quad (34)$$

The coefficient counts the ways to choose the target option (χ, z), the last explored option $(\mathcal{X}_n, \mathcal{Z}_n)$, and the $n-2$ prior options $(\mathcal{X}', \mathcal{Z}')$, and $N-n$ later options $(\mathcal{X}'', \mathcal{Z}'')$.

First, the density of $w \equiv \chi + z$ is $\int_s^\infty h(\zeta(c) - t)\, g(t + w - \zeta(c))dt$, where $\chi_n = s - \zeta(c) + w$. Event $\mathcal{X}_n + \zeta(c) > w$ in $(30')$ has density $g(s + w - \zeta(c))$ for $s > 0$, and given χ_n, the second inequality event in $(30')$ has chance $P(\mathcal{Z}_n < w - \chi_n) = H(\zeta(c) - s)$. Each of the $n-2$ events in $(30'')$ has chance $\int_s^\infty H(\zeta(c) - t)g(t + w - \zeta(c))dt = \iota(s, w - \zeta(c), \zeta(c))$, if $\iota(s, \chi, \zeta(c)) \equiv \int_s^\infty H(\zeta(c) - t)g(t + \chi)dt$. By event independence:

$$\Upsilon(w) \equiv \int_0^\infty H(\zeta(c) - s)g(s + w - \zeta(c))\left[\int_s^\infty h(\zeta(c) - t)g(t + w - \zeta(c))dt\right]\iota(s, w - \zeta(c), \zeta(c))^{n-2}ds.$$

Recalling (7) and (34), then (20) and (21), the stage-n conditional recall chance equals

$$\mathcal{R}_n = \frac{r_n}{\sigma_n} = \frac{(n-1)\int_{u-\zeta(c)}^\infty \Upsilon(\chi + \zeta(c))G(\chi)^{N-n}d\chi}{\int_{u-\zeta(c)}^\infty g(\chi)\delta(\chi, c)^{n-1}G(\chi)^{N-n}dx} = E_{\mathcal{X}_n}\left[\frac{(n-1)\Upsilon(\mathcal{X}_n + \zeta(c))}{g(\mathcal{X}_n)\delta(\mathcal{X}_n, c)^{n-1}}\right] \quad (35)$$

where $\chi = w - \zeta(c)$. Since $\mathcal{R}_n = E_{\mathcal{X}_n}[B(\mathcal{X}_n, n)]$ by Claim 3, the $B(\mathcal{X}_n, n)$ formula (32) is the bracketed term in (35). Since $B(\chi, n) = (n-1)\Upsilon(\chi + \zeta(c))/[g(\chi)\delta(\chi, c)^{n-1}]$,

$$\frac{B(\chi, n)}{n-1} = \int_0^\infty \frac{H(\zeta(c) - s)g(s + \chi)}{g(\chi)}\left[\frac{\int_s^\infty h(\zeta(c) - t)\, g(t + \chi)dt}{\int_s^\infty H(\zeta(c) - t)g(t + \chi)dt}\right]\nu(s, \chi, \zeta(c))^{n-1}ds \quad (36)$$

where we define $\nu(s, \chi, \zeta(c)) \equiv \iota(s, \chi, \zeta(c))/\delta(\chi, c)$.

We argue that $B(\chi, n)$ falls in χ: First, $g(s + \chi)/g(\chi)$ falls in χ, as g is log-concave. Second, given g and H log-concave, $\mathbb{I}_{t \geq s}H(\zeta(c) - t)g(t + \chi)$ is log-supermodular in $(s, \zeta(c), -\chi, t)$, the integral $\iota(s, \chi, \zeta(c)) \equiv \int_s^\infty H(\zeta(c) - t)g(t + \chi)dt$ is log-supermodular in $(\zeta(c), -\chi)$, by Karlin and Rinott (1980), and so log-submodular in $(\zeta(c), \chi)$. Hence,

$$\frac{\int_s^\infty h(\zeta(c) - t)\, g(t + \chi)dt}{\int_s^\infty H(\zeta(c) - t)g(t + \chi)dt} = \frac{\partial \log[\iota(s, \chi, \zeta)]}{\partial \zeta} \quad (37)$$

falls in χ. By log-concavity of g, the following also falls in χ:

$$\frac{\partial \log[\iota(s, \chi, \zeta(c))]}{\partial s} = \frac{-H(\zeta(c) - s)g(s + \chi)}{\int_s^\infty H(\zeta(c) - t)g(t + \chi)dt} = \frac{-H(\zeta(c) - s)}{\int_0^\infty H(\zeta(c) - r - s)\frac{g(r + s + \chi)}{g(s + \chi)}dr} \quad (38)$$

So $\iota(s, \chi, \zeta(c))$ is log-submodular in (s, χ), and $\nu(s, \chi, \zeta(c)) \equiv \iota(s, \chi, \zeta(c))/\iota(0, \chi, \zeta(c))$ falls in χ. As $g(s + \chi)/g(\chi)$, and (37) and (38) fall in χ, $B(\chi, n)$ in (36) falls in χ. $\square$

C Search Duration and Prize Dispersion Proofs

C.1 Proof of Increased Search Duration Claim in Theorem 4

Index the distribution H_t so that $\mathcal{Z}_t$ has a mean-enhancing dispersion as t rises. The quantile function steepens in t, or $H_t^{-1}(\bar{\alpha})-H_t^{-1}(\alpha)$ rises in t if $\bar{\alpha} > \alpha$; if differentiable, $\partial H_t^{-1}(\alpha)/\partial\alpha$ rises in t. Let $\zeta_t(c)$ solve the analogous Bellman equation (3).

Claim 5 *For any $\Delta \geq 0$, $H_t(\zeta_t(c) - \Delta)$ increases in the dispersion index t.*

Proof: Changing variables from z to $\alpha = H_t(z-\Delta)$ in the Bellman equation (3) yields:

$$c = \int_{H_t(\zeta_t(c)-\Delta)}^{1} (1 - H_t(H_t^{-1}(\alpha) + \Delta))\frac{\partial H_t^{-1}(\alpha)}{\partial \alpha} d\alpha \tag{39}$$

Put $A_t(\alpha, \Delta) \equiv H_t(H_t^{-1}(\alpha)+\Delta)$. So $H_t^{-1}(A_t(\alpha, \Delta))-H_t^{-1}(\alpha) \equiv \Delta$, and $A_t(\alpha, \Delta)$ falls in t (more disperse). As t rises, the integrand of (39) rises, as does $H_t(\zeta_t(c) - \Delta)$.□

Claim 6 *Every survival chance σ_n rises in the dispersion index t.*

Proof: By (7), the survival chance σ_n rises in $\delta_t(\chi, c)$ and falls in $u-\zeta_t(c)$. By Claim 5, $H_t(\zeta_t(c) - s)$ rises in t. From (5), $\delta_t(\chi, c) = \int_0^\infty H_t(\zeta_t(c) - s)g(x + s)ds$ rises in t, by (5). Thus, σ_n rises if $u - \zeta_t(c)$ falls in t. Now, a mean-enhancing dispersion in $\mathcal{Z}_t$ is a mean-preserving dispersion of $\mathcal{Z}_t$ plus a positive constant. Also, $\zeta_t(c)$ rises in any MPS of $\mathcal{Z}_t$ by (3), and thus in a mean-preserving dispersion. Also, $\zeta_t(c)$ rises when $\mathcal{Z}_t$ shifts up by a positive constant, by (3). So $\zeta_t(c)$ rises in t, and $u - \zeta_t(c)$ falls. □

C.2 Proof of the Recall Moment Claim in Theorems 4 and 9

The chance ρ_n that the recall moment is at least stage $n \geq 1$ falls in n. By Lemma 1, Sam strikes or passes in stage n if the fallback is at most the cutoff $\mathcal{X}_{n+1} + \zeta(c)$. So

$$\begin{aligned}\rho_n &= P(\max_{j\leq n-1}\{u, \mathcal{X}_j + \mathcal{Z}_j\} < \mathcal{X}_{n+1} + \zeta(c)). \qquad (40)\\ &= N\binom{N-1}{n-1}\int_{u-\zeta(c)}^{\infty}\int_{x_{n+1}}^{\infty}\left[\int_{x_n}^{\infty} H(x_{n+1} + \zeta(c) - x)dG(x)\right]^{n-1} dG(x_n)dG(x_{n+1})^{N-n}\end{aligned}$$

Claim 7 *The recall moment rises with a mean-preserving dispersion increase of $\mathcal{Z}$.*

Proof: By Claim 5, $H(x_{n+1}+\zeta(c)-x)$ increases in the dispersion of $\mathcal{Z}$. Also, the optionality value $\zeta(c)$ increases as $\mathcal{Z}$ incurs a mean-preserving dispersion, reducing the lower support $u-\zeta(c)$ of the integral in (40). Altogether, ρ_n increases. □

Claim 8 *The recall moment stochastically increases in the number of options N.*

Proof: By the Markov property of order statistics, the joint distribution of the n top known factors is that of n i.i.d. draws $\mathcal{X}$ from G, given $\mathcal{X} > x_{n+1}$ (proof of Theorem 2), i.e., with cdf $\tilde{G}(x) = G(x)/[1-G(x_{n+1})]$ on $[x_{n+1},\infty)$. Since $1-G$ is log-concave, $\tilde{G}(x)$ grows less disperse as x_{n+1} increases (Theorem 3.B.19 in Shaked and Shanthikumar (2007)). So the gap between draws from $\tilde{G}$ and x_{n+1} stochastically shrinks. As N rises, $u-\mathcal{X}_{n+1}$ and $\mathcal{X}_j-\mathcal{X}_{n+1}$ stochastically fall, and so ρ_n rises, as (40) implies:

$$\rho_n = P(\max_{j\le n-1}\{u-\mathcal{X}_{n+1}, \mathcal{X}_j-\mathcal{X}_{n+1}+\mathcal{Z}_j\} < \zeta(c)) \qquad \square$$

C.3 Dispersion in $\mathcal{X}$: Proof of Theorem 5 and Corollary 1

Theorem 5: Write $\mathcal{X}_j-\mathcal{X}_n = \Sigma_{k=j}^{n-1}(\mathcal{X}_k-\mathcal{X}_{k+1}) = \Sigma_{k=j}^{n-1}\Delta_k$, for $\Delta_k \equiv \mathcal{X}_k-\mathcal{X}_{k+1}\ge 0$. Then

$$\sigma_n = P(\{\mathcal{X}_n+\zeta(c) > \Omega_{n-1}) = P\left(\{\mathcal{X}_n+\zeta(c)>u\}\cap_{j<n}\left\{\zeta(c)-\Sigma_{k=j}^{n-1}\Delta_k \ge \mathcal{Z}_j\right\}\right)$$

is the chance of event (6). By the joint distribution ψ of $\vec{\Delta}_n=(\Delta_1,\ldots,\Delta_{n-1})$ and $\mathcal{X}_n$:

$$\sigma_n = \int_{x_n\in\mathbb{R},\vec{\Delta}_n\in\mathbb{R}_+^{n-1}} \mathbb{I}_{\{x_n+\zeta(c)\ge u\}}\prod_{j=1}^{n-1} H(\zeta(c)-\Sigma_{k=j}^{n-1}\Delta_k)d\psi(\vec{\Delta}_n, x_n). \qquad (41)$$

With no quit payoff (u=$-\infty$), the indicator $\mathbb{I}$=1 in (41). As $\mathcal{X}$ grows less dispersive, quantiles compress by (10), and $\vec{\Delta}_n \equiv \{\mathcal{X}_1-\mathcal{X}_2,\ldots,\mathcal{X}_j-\mathcal{X}_{j+1},\ldots,\mathcal{X}_{n-1}-\mathcal{X}_n\}$ falls stochastically. Since all gaps Δ_j stochastically fall for j=$1,\ldots,n-1$, σ_n rises. □

Corollary 1: Let $\tau(u,\mathcal{X})$ be the search duration. We argue that $\tau(u,\mathcal{X})<\tau(u,0)$ iff $\zeta(c)>u$. Assume $\zeta(c)>u$. First, $\tau(-\infty,\mathcal{X})<\tau(-\infty,0)$ for nondegenerate $\mathcal{X}$ (Theorem 5). As noted after (7), search duration falls in u, for non-degenerate $\mathcal{X}$, i.e. $\tau(u,\mathcal{X})<\tau(-\infty,\mathcal{X})$ for all $u>-\infty$. But when $\mathcal{X}=0$, Sam never stops if $u<\zeta(c)$. So search duration is constant in u: $\tau(-\infty,0)=\tau(u,0)$. So $\tau(u,\mathcal{X})<\tau(-\infty,\mathcal{X})<\tau(-\infty,0)=\tau(u,0)$. But if $\zeta(c)\le u$ then search duration is $\tau(u,\mathcal{X})>\tau(u,0)=0$. □

D Web Search Proofs

D.1 Expected Search Duration and Attraction

Claim 9 (Optionality) *The search optionality value $\zeta(\alpha, c)$ falls in accuracy α, when $\zeta(0, c) > -c = \zeta(1, c)$. Also, $\zeta(\alpha, c)/\sqrt{1-\alpha^2}$ monotonically falls in α to $-\infty$.*

Proof: As α rises, the hidden factor experiences a mean preserving contraction, and so $\zeta(\alpha, c)$ falls. As $\alpha \uparrow 1$, (13) reduces to $c = \int_{\zeta(\alpha,c)}^{0} ds = -\zeta(\alpha, c)$, and so $\zeta(\alpha, c) \downarrow -c$.

Next, change variables to $s' = s/\sqrt{1-\alpha^2}$ in (13), and let $z(\alpha) \equiv \zeta(\alpha, c)/\sqrt{1-\alpha^2}$. This yields $c/\sqrt{1-\alpha^2} = \int_{z(\alpha)}^{\infty} [1 - \Phi(s')]\, ds'$. The LHS rises to ∞ as α rises to 1. Since the mean of a left truncated Gaussian distribution is finite, $\int_{z(\alpha)}^{\infty} [1 - \Phi(s')]\, ds' = E[\max\{z(\alpha), S'\}] - z(\alpha)$ is finite if $z(\alpha) > -\infty$. So $z(\alpha) \downarrow -\infty$ as α rises to 1. □

Attraction and Expected Search Duration. The *attraction* is the chance that a random option $(\mathcal{X}, \mathcal{Z})$ does not prevent Sam from exploring an option with known factor χ, i.e. the sum of the chance that $\chi > \mathcal{X}$ and the delay chance $\delta(\chi, c)$, or, $\pi(\alpha, x) \equiv G(\chi) + \delta(\chi, c)$. Put $\mathcal{X} = \alpha X$ and $\mathcal{Z} = \sqrt{1-\alpha^2} Z$ into the δ formula in (5). Then

$$\pi(\alpha, x|c) = \int_0^{\infty} \Phi\left(\frac{\zeta(\alpha, c) - \alpha s}{\sqrt{1-\alpha^2}}\right) \phi(x+s) ds + \Phi(x). \tag{42}$$

Next, using (7), the expected search time formula $\tau(\alpha) = \sum_{n=1}^{N} \sigma_n$ collapses to

$$\tau(\alpha) = N \int_{\ell(\alpha,u,c)}^{\infty} \pi(\alpha, x|c)^{N-1} \phi(x) dx. \tag{43}$$

D.2 Rising Accuracy, I: Proof of Theorem 6

Claim about τ: If $u = -\infty$, then $\tau(\alpha) = N \int_{-\infty}^{\infty} \pi(\alpha, x|c)^{N-1} d\Phi(\chi)$ by (43), as $\ell(\alpha, -\infty, c) = -\infty$ by (14). Then $\tau'(\alpha) < 0$, since $\pi_\alpha(\alpha, x) < 0$ by Claim 10 in §D.3.

Claim about $E[\mathcal{W}^A(\alpha, c)]$: We argue $\frac{\partial}{\partial \alpha} E[\mathcal{W}^A(\alpha, c)] < 0$ for small $c > 0$ and for all u. By footnote 30, we only need show $\mathcal{M}_c(\alpha, c) \downarrow 0$ as $c \to 0$. By (16) and (43),

$$\begin{aligned}\mathcal{M}_c(\alpha, c) = -\tau'(\alpha) = &- N(N-1) \int_{\ell(\alpha,u,c)} \pi(\alpha, x|c)^{N-2} \frac{\partial \pi(\alpha, x|c)}{\partial \alpha} d\Phi(\chi) \\ &+ N \ell_\alpha(\alpha, u, c) \phi(\ell(\alpha, u|c)) \pi(\alpha, \ell|c)^{N-1}.\end{aligned} \tag{44}$$

Let $c \downarrow 0$. Then the first line of (44)'s RHS vanishes. For $\zeta(\alpha, c) \uparrow \infty$ by (13) and so $\pi(\alpha, x|c) \to 1$ by (42), and $\pi(\alpha, x|c)^{N-2} \to 1$ in (44). Differentiation of (13) yields

$$\zeta_\alpha(\alpha, c) = -\frac{\alpha}{\sqrt{1-\alpha^2}} \frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))} < 0. \tag{45}$$

where $z(\alpha) \equiv \zeta(\alpha, c)/\sqrt{1-\alpha^2}$. Then by (42) and (45),

$$\frac{\partial \pi(\alpha, x|c)}{\partial \alpha} = -\int_0^\infty \phi\left(z(\alpha) - \frac{\alpha}{\sqrt{1-\alpha^2}} s\right) \phi(\chi + s) \left(\frac{\alpha\left(\frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))} - z(\alpha)\right)}{1-\alpha^2} + \frac{s}{(1-\alpha^2)^{3/2}} \right) ds$$

Since $z(\alpha) \uparrow \infty$ as $c \downarrow 0$, and so $\phi(z(\alpha) - \alpha s/\sqrt{1-\alpha^2}) \to 0$. By the inverse Mills ratio, $\phi(z(\alpha))/[1-\Phi(z(\alpha))] - z(\alpha) = E[Z - z(\alpha)|Z > z(\alpha)] > 0$ falls in $z(\alpha)$, by log-concavity. So $\frac{\partial}{\partial \alpha}\pi(\alpha, x|c) \to 0$ as $c \downarrow 0$, and thus so does the first line of (44).

Put $\ell \equiv \ell(\alpha, c) \equiv \ell(\alpha, u, c)$. The second line of (44) vanishes as $c \downarrow 0$, since $\ell_\alpha(\alpha, c)\phi(\ell(\alpha, c)) \to 0$. Put (45) into $\ell_\alpha(\alpha, c) = -[\ell(\alpha, c) + \zeta_\alpha(\alpha, c)]/\alpha$ (footnote 33):

$$\begin{aligned} \ell_\alpha(\alpha, c)\phi(\ell) &= -\frac{\phi(\ell)}{\alpha}\left[\ell(\alpha) - \frac{\alpha}{\sqrt{1-\alpha^2}} \frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))}\right] \\ &= -\frac{\phi(\ell)\ell}{\alpha}\left[1 - \frac{\alpha}{\sqrt{1-\alpha^2}} \frac{z(\alpha)}{\ell} - \frac{\alpha}{\ell\sqrt{1-\alpha^2}}\left(\frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))} - z(\alpha)\right)\right] \end{aligned} \tag{46}$$

To show that (46) vanishes as $c \downarrow 0$, we add and subtract $z(\alpha)\alpha/\sqrt{1-\alpha^2}$ to the bracketed term, and factor out ℓ. As $c \downarrow 0$, $\zeta(\alpha, c) \uparrow \infty$ by (42). So by (14), $\ell(\alpha, c) \equiv [u - \zeta(\alpha, c)]/\alpha \downarrow -\infty$, and thus the lead factor on (46) vanishes: $\phi(\ell)\ell \to 0$, for ϕ a Gaussian density. The bracketed term is boundedly finite, as $z(\alpha) \equiv \zeta(\alpha, c)/\sqrt{1-\alpha^2}$ implies $z(\alpha)/\ell(\alpha, c) \to -\alpha/\sqrt{1-\alpha^2}$ as $\zeta \uparrow \infty$. The last parenthetical term equals $E[Z - z(\alpha)|Z > z(\alpha)] > 0$, by the inverse Mills ratio, and falls to 0 as $z(\alpha) \uparrow \infty$. □

D.3 Increasing Accuracy, II: Proof of Theorem 7

Claim about quitting chance: Recall the attraction notion in §D.1. Given $\pi(\alpha, \ell(\alpha, u, c)|c) \equiv \delta(u - \zeta(\alpha, c), c) + G(u - \zeta(\alpha, c))$, $q(\alpha) = \Sigma_{n=1}^N q_n$, and formula (24), the chance that Sam either does not search, or does, but *eventually quits*, equals:

$$q(\alpha) = \Sigma_{n=0}^N \binom{N}{n} \delta(u - \zeta(\alpha, c), c)^n G(u - \zeta(\alpha, c))^{N-n} = \pi(\alpha, \ell(\alpha, u, c)|c)^N. \tag{47}$$

By algebraic simplification (see Online Appendix II):

$$\frac{\partial}{\partial\alpha}\pi(\alpha,\ell(\alpha,u,c)|c) = -\left[1-\Phi\left(\zeta(\alpha,c)/\sqrt{1-\alpha^2}\right)\right]\ell(\alpha,u,c)\phi(\ell(\alpha,u,c))/\alpha. \quad (48)$$

We have $q'(\alpha) > 0$ iff $\partial\pi(\alpha,\ell(\alpha,u,c))/\partial\alpha > 0$, and so by (48), iff $\ell(\alpha,u,c) < 0$, or $\zeta(\alpha,c) > u$. Given Claim 9, this validates Figure 8 and proves the statement for q.

Claim about CTR: The CTR is $\sigma_1 = 1-\Phi(\ell(\alpha,u,c))^N$ and hence $\partial\sigma_1/\partial\alpha = -N\phi(\ell)\Phi(\ell)^{N-1}\ell_\alpha$ has the sign of $-\ell_\alpha(\alpha,u,c)$. The derivative $\ell_\alpha(\alpha,u,c)$ falls in u by footnote 33, and $\ell_\alpha(\alpha,u,c) > 0$ for low enough u, by (14). So the CTR claims holds.

Claim about τ: We show that τ_α is single-crossing in u, i.e., negative then positive as u increases. Since $\pi_\alpha(\alpha,x|c) < 0$ by Claim 10 below, and $\ell_\alpha(\alpha,u,c) > 0$ for $\ell < 0$, we have $\tau_\alpha < 0$ for low enough u, from (43). Differentiate (43) in α, and then change variables $s = x-\ell(\alpha,u,c)$. Writing $\ell(\alpha,u,c) = \ell$, the slope $\partial\tau/\partial\alpha$ equals:

$$\begin{aligned} & -\ell_\alpha N\pi(\alpha,\ell|c)^{N-1}\phi(\ell) + N(N-1)\int_0^\infty \pi(\alpha,\ell+s|c)^{N-2}\pi_\alpha(\alpha,\ell+s)\phi(\ell+s)ds \quad (49) \\ &= N\pi(\alpha,\ell)^{N-1}\phi(\ell)\left[-\ell_\alpha + (N-1)\int_0^\infty\left[\frac{\pi(\alpha,\ell+s|c)}{\pi(\alpha,\ell|c)}\right]^{N-1}\frac{\pi_\alpha(\alpha,\ell+s|c)}{\pi(\alpha,\ell+s|c)}\frac{\phi(\ell+s)}{\phi(\ell)}ds\right] \end{aligned}$$

Let $\Xi(\alpha,u)$ be the bracketed term. Pick large u so that $\ell = \ell(\alpha,u,c) \geq 0$. The integrand in Ξ is negative and rises in ℓ, and thus in u given $\ell_u(\alpha,u,c) > 0$ by (14):

- $0 > \dfrac{\pi_\alpha(\alpha,\ell+s|c)}{\pi(\alpha,\ell+s|c)}$ rises in ℓ by log-supermodularity of $\pi(\alpha,x)$ (Claim 10 below)
- $\dfrac{\pi(\alpha,\ell+s|c)}{\pi(\alpha,\ell|c)}$ falls in ℓ by log-concavity of $\pi(\alpha,x|c)$ in x (Claim 11 below)
- The last fraction $\phi(\ell+s)/\phi(\ell)$ falls in ℓ, since ϕ is strictly log-concave.

Also, $\ell_\alpha(\alpha,u,c)$ falls in u by (14). As $\Xi(\alpha,u)$ is increasing in u, $\tau_\alpha(\alpha,u)$ is strictly single-crossing in u. But as noted above, $\tau_\alpha(\alpha,u) < 0$ for small enough u. When $u\to\infty$, $\ell_\alpha(\alpha,u,c)\to-\infty$ and so $\Xi(\alpha,u)>0$, and $\tau_\alpha(\alpha,u)>0$. So $\tau_\alpha(\alpha,u)$ changes sign exactly once as u rises from $-\infty$ to ∞. □

Claim 10 (Log-supermodularity of Attraction) $\pi_\alpha(\alpha,x) < 0 < (\log[\pi(\alpha,x)])_{x\alpha}$.

Proof: Since $\partial[\zeta(\alpha,c)/\sqrt{1-\alpha^2}]/\partial\alpha < 0$ by Claim 9, differentiating (42) yields $\pi_\alpha < 0$. Next, rewrite (42) as $\pi(\alpha,x) = \int_{-\infty}^{\infty} \phi(x+s)f(\alpha,s)ds$, where for $s \leq 0$:

$$f(\alpha,s) \equiv \Phi\left((\zeta(\alpha,c) - \alpha s)/\sqrt{1-\alpha^2}\right)$$

and $f(\alpha,s) \equiv 1$ for $s \leq 0$. Since the Gaussian density obeys $\phi'(x) = -x\phi(x)$, we have

$$-\frac{\partial \log[\pi(\alpha,x)]}{\partial x} = \frac{\int_{-\infty}^{\infty}(x+s)\phi(x+s)f(\alpha,s)ds}{\int_{-\infty}^{\infty}\phi(x+s)f(\alpha,s)ds} = \frac{\int_{-\infty}^{\infty} r\phi(r)f(\alpha,r-x)dr}{\int_{-\infty}^{\infty}\phi(r)f(\alpha,r-x)dr}. \quad (50)$$

This is the mean of a r.v. with density $\phi(r)f(\alpha,r-x)$. Next, we argue $f(\alpha,s)$ is log-submodular, or equivalently $f(\alpha,s_2)/f(\alpha,s_1)$ falls in α for all $s_2 > s_1$. First, if $s_1, s_2 < 0$, we have $f(\alpha,s_2)/f(\alpha,s_1) \equiv 1/1 = 1$ weakly falls in α. Second, if $s_1, s_2 > 0$, then $f(\alpha,s_i) \equiv \Phi((\zeta(\alpha,c)-\alpha s_i)/\sqrt{1-\alpha^2})$ for $i = 1,2$. It suffices that this falls in α:

$$\frac{\partial \log[f(\alpha,s)]}{\partial s} = -\frac{\alpha s}{\sqrt{1-\alpha^2}}\phi\left(\frac{\zeta(\alpha,c)-\alpha s}{\sqrt{1-\alpha^2}}\right) \Big/ \Phi\left(\frac{\zeta(\alpha,c)-\alpha s}{\sqrt{1-\alpha^2}}\right). \quad (51)$$

This follows from Φ log-concave, and because $[\zeta(\alpha,c)-\alpha s]/\sqrt{1-\alpha^2}$ falls in α, as $\partial[\zeta(\alpha,c)/\sqrt{1-\alpha^2}]/\partial\alpha < 0$ by Claim 9 and $s > 0$. Third, $f(\alpha,s_2)/f(\alpha,s_1) = f(\alpha,s_2) = \Phi\left(\frac{\zeta(\alpha,c)-\alpha s_2}{\sqrt{1-\alpha^2}}\right)$ falls in α, for $s_1 \leq 0 < s_2$. Altogether $f(\alpha,s)$ is log-submodular, and thus the middle term in (50) falls in α, or $\partial^2 \log[\pi(\alpha,x)]/\partial\alpha\partial x > 0$. □

Claim 11 (Log-Concavity of Attraction) $\pi_x(\alpha,x) > 0 > (\log[\pi(\alpha,x)])_{xx}$ *if* $x \geq 0$

Proof: For the log-concavity of $\pi(\alpha,x)$ in x, integrate (42) by parts to get

$$\pi(\alpha,x) = \frac{\alpha}{\sqrt{1-\alpha^2}}\int_0^\infty \phi\left(\frac{\zeta(\alpha,c)-\alpha s}{\sqrt{1-\alpha^2}}\right)\Phi(x+s)ds + \Phi(x)\left(1-\Phi\left(\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}\right)\right).$$

Then

$$\pi_x(\alpha,x) = \frac{\alpha}{\sqrt{1-\alpha^2}}\int_0^\infty \phi\left(\frac{\zeta(\alpha,c)-\alpha s}{\sqrt{1-\alpha^2}}\right)\phi(x+s)ds + \phi(x)\left(1-\Phi\left(\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}\right)\right) > 0$$

Since the Gaussian density ϕ is hump-shaped and peaks at 0, the RHS falls in $x \geq 0$. In other words, $(\log[\pi(\alpha,x)])_{xx} < 0$ for $x \geq 0$. □

D.4 Marginal Value of Information: Proof of Lemma 5

By definition, the $\mathcal{L}$ event occurs when Sam explores an option and draws $Z^A \geq z(\alpha) \equiv \zeta(\alpha, c)/\sqrt{1-\alpha^2}$. Given the logic in (6a)-(6c), Sam explores an option with known factor χ if it dominates the outside option, i.e. $\chi > \ell(\alpha, u, c)$, and the other options either ranked after χ or delay it, which happens with chance $\pi(\chi, c)^{N-1}$. Thus, the $\mathcal{L}$ event has ex-ante chance $P(\mathcal{L}) = [1 - \Phi(z(\alpha))]N \int_{\ell(\alpha,u,c)}^{\infty} \pi(\chi, c)^{N-1} d\Phi(x)$ and $P(\mathcal{L})E[X^A|\mathcal{L}]$ can be written as the right side of (52) below:

Claim 12 *The marginal value of information can be written as*

$$\mathcal{M}(\alpha, c, u) = \frac{\partial \mathcal{V}(\alpha, c, u)}{\partial \alpha} = [1 - \Phi(z(\alpha))]\, N \int_{\ell(\alpha,u,c)}^{\infty} \pi(\alpha, x)^{N-1} x d\Phi(x) \geq 0. \quad (52)$$

Proof: First, $\partial \mathcal{V}/\partial u = q$, by the proof of Claim 22. Hence, $\partial^2 \mathcal{V}/\partial u \partial \alpha = q'(\alpha)$. Since the quitting chance is $q(\alpha) = \pi(\alpha, \ell(\alpha, u, c))^N$ by (47), then by (48),

$$\frac{\partial q}{\partial \alpha} = N\pi(\alpha, \ell(\alpha, u, c))^{N-1} \left(\frac{-u + \zeta(\alpha, c)}{\alpha^2}\right) \phi \left(\frac{u - \zeta(\alpha, c)}{\alpha}\right) \left[1 - \Phi\left(\frac{\zeta(\alpha, c)}{\sqrt{1-\alpha^2}}\right)\right]. \quad (53)$$

We can derive $\partial \mathcal{V}/\partial \alpha$ by integrating $\partial q/\partial \alpha$ over u, using (53). We use the boundary condition $\partial \mathcal{V}/\partial \alpha|_{u=\infty} = 0$, because as $u \to \infty$ Sam never searches through and the accuracy is payoff irrelevant. Integrating (53) in u yields (52).

As $0 < \pi(\alpha, x)^{N-1}$ is strictly increasing in x, by Claim 11 , and $\int_{\ell(\alpha,u,c)}^{\infty} x d\Phi(x) \geq 0$, the integral in (52) is positive.[39] Hence, $\mathcal{M}(\alpha, c, u) \geq 0$. □

D.5 Marginal Value of Information Proofs: Theorem 8, Etc.

Claim that $\mathcal{M}(1, c, u) > \mathcal{M}(0, c, u)$. We prove it in each of three quit payoff u cases:

Case 1: $u > \zeta(0, c)$: As $\alpha \to 0$, by (14), $\ell(\alpha, u, c) \to \infty$ if $u > \zeta(0, c)$, $\ell(\alpha, u, c) \to -\infty$ if $u < \zeta(0, c)$, and $\ell(\alpha, u, c) \to 0$ if $u = \zeta(0, c)$. So $\mathcal{M}(0, c, u) = 0$ if $u > \zeta(0, c)$, by (52). Next, recall $z(\alpha) \equiv \zeta(\alpha, c)/\sqrt{1-\alpha^2}$. As $\alpha \to 1$, by Claim 9, $z(\alpha) \to -\infty$. Thus, $\pi(\alpha, x) \to \Phi(x)$ by (42). Since $\ell \to u - c$ by (14) and Claim 9, $\mathcal{M}(1, c, u) > 0$ by (52). Hence, $\mathcal{M}(1, c, u) > \mathcal{M}(0, c, u)$ for $u > \zeta(0, c)$.

[39] By Karlin and Rubin (1955), if f is single-crossing and $\int f(x)dx \geq 0$, positivity is preserved if one multiplies the integrand by a positive and increasing function $b(x)$, namely $\int f(x)b(x)dx \geq 0$.

CASE 2: $u = -\infty$: Divide and multiply (52) by $\tau(\alpha)$, where $\tau(\alpha)$ is given by (43), then:

$$\mathcal{M}(\alpha, c, u) = \frac{\int_{\ell(\alpha,u,c)}^{\infty} \pi(\alpha, x)^{N-1} x d\Phi(x)}{\int_{\ell(\alpha,u,c)}^{\infty} \pi(\alpha, x)^{N-1} d\Phi(x)} \tau(\alpha) \left[1 - \Phi\left(z(\alpha)\right)\right]. \tag{54}$$

When $u = -\infty$, $\ell(\alpha, u, c) = -\infty$. Hence, the fraction on the RHS of (54) rises in α because $\pi(\alpha, x)$ is log-supermodular in (α, x) by Claim 10. So this fraction is higher when $\alpha = 1$ than when $\alpha = 0$, and strictly so when $N > 1$. When $N = 1$, the fraction vanishes and thus $\mathcal{M}(0, c, -\infty) = \mathcal{M}(1, c, -\infty) = 0$.

We show $\tau\left[1-\Phi\left(z(\alpha)\right)\right]$ is higher $\alpha = 1$ than at $\alpha = 0$. As $\alpha \to 1$, $\tau \to 1$, as Sam always participates when $u = -\infty$, and one search suffices as $\alpha \to 1$. Also $\left[1-\Phi\left(z(\alpha)\right)\right] \to 1$ since $z(\alpha) \to -\infty$, by Claim 9. Then $\tau\left[1-\Phi\left(z(\alpha)\right)\right] \to 1$ as $\alpha \to 1$. As $\alpha \to 0$, Sam stops iff $Z > z(\alpha)$, except in the last period. So the search duration $\tau(\alpha)$ is strictly lower than that of an infinite horizon stationary model with constant cutoff $z(\alpha)$, which is $1/\left[1-\Phi\left(z(\alpha)\right)\right]$ by standard results. So $\tau\left[1-\Phi\left(z(\alpha)\right)\right] < 1$ as $\alpha \to 0$. So $\mathcal{M}(0, c, -\infty) \leq \mathcal{M}(1, c, -\infty)$ and the inequality binds iff $N = 1$.

CASE 3: $u \in (-\infty, \zeta(0, c)]$: By (52), $\mathcal{M}(\alpha, c, u)$ is hump-shaped in u and peaks at $u = \zeta(\alpha, c)$. Since $\zeta(1, c) < \zeta(0, c)$, $\mathcal{M}(1, c, u)$ rises and then falls as u rises in $(-\infty, \zeta(0, c)]$. In the same interval of u, as $\alpha \to 0$, by (14), $\ell(\alpha, u, c) \to -\infty$ if $u < \zeta(0, c)$, and $\ell(\alpha, u, c) \to 0$ if $u = \zeta(0, c)$. Hence $\mathcal{M}(0, c, u)$ is constant in u for $u \in (-\infty, \zeta(0, c))$ and jumps up at $u = \zeta(0, c)$ by (52). Since we showed $\mathcal{M}(0, c, u) \leq \mathcal{M}(1, c, u)$ at $u = -\infty$, we need only check the inequality at $u = \zeta(0, c)$.

When $u = \zeta(0, c)$, $\ell(\alpha, u, c) \to 0$ as $\alpha \to 0$. Also, $\pi(0, x) = \Phi(x) + [1-\Phi(x)]\Phi\left(\zeta(0, c)\right)$ by (42), as $\alpha \to 0$. Hence, $\partial\pi(0, x)/\partial x = \phi(x)[1 - \Phi\left(\zeta(0, c)\right)]$. Given (43), rewrite (54) as

$$\mathcal{M}(0, c, \zeta(0, c)) = \frac{\int_{0}^{\infty} \pi(0, x)^{N-1} x d\Phi(x)}{\int_{0}^{\infty} \pi(0, x)^{N-1} d\Phi(x)} \int_{0}^{\infty} N\pi(0, x)^{N-1} \frac{\partial \pi(0, x)}{\partial x} dx \tag{55}$$

where the last integral equals $1 - \pi(0, 0)^N$.

As $\alpha \to 1$, $z(\alpha) \to -\infty$ and so $[1 - \Phi(z(\alpha)] \to 1$. Since Sam explores at most one option, search duration is his participation chance: $\tau(\alpha) = \sigma_1(\alpha) = 1-\Phi(\ell(\alpha, u, c))^N$. Also, $\zeta(\alpha, c) \to -c$ and so $\ell(\alpha, \zeta(0, c), c) \to \zeta(0, c) + c$. Hence (54) becomes

$$\mathcal{M}(1, c, \zeta(0, c)) = \frac{\int_{\zeta(0,c)+c}^{\infty} \pi(1, x)^{N-1} x d\Phi(x)}{\int_{\zeta(0,c)+c}^{\infty} \pi(1, x)^{N-1} d\Phi(x)} [1 - \Phi(\zeta(0, c) + c)^N]. \tag{56}$$

We first argue the fraction in (56) exceeds that in (55). The fraction in (54) is the truncated mean of a r.v. with density $\pi(\alpha,x)^{N-1}\phi(x)$ and support $[\ell,\infty)$. Fixing ℓ, it rises in α since $\pi(\alpha,x)$ is log-supermodular in (α,x) by Claim 10. This fraction rises in the lower support ℓ, as a truncated mean. The lower support of the truncated mean in (55) is 0 and that in (56) is $\zeta(0,c)+c$. We have $\zeta(0,c)+c>0$ because $\zeta(0,c)>\zeta(1,c)=-c$ (Claim 9). Hence, the fraction in (56) exceeds that in (55). Finally, the last integral in (55) is $1-\pi(0,0)^N$ and is smaller than $1-\Phi(\zeta(0,c)+c)^N$ in (56) because $\pi(0,0)>\Phi(\zeta(0,c)+c)$ ((60) in Online Appendix III).

Claim that $\mathcal{M}_\alpha(\alpha,c)>0$ for small search costs $c>0$: Recall that $\zeta(\alpha,c)$ falls in α, and $\zeta(\alpha,c)\uparrow\infty$ as $c\downarrow 0$. So $\ell(\alpha,u,c)\equiv[u-\zeta(\alpha,c)]/\alpha$ rises in α for small $c>0$. So the fraction in the RHS of (54) rises in α for small $c>0$ because $\pi(\alpha,\chi)$ is log-supermodular and $\ell_\alpha(\alpha,c)>0$ for small $c>0$. Since by (45),

$$z'(\alpha)=\frac{\partial}{\partial\alpha}\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}=-\frac{\alpha}{1-\alpha^2}\left(\frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))}-z(\alpha)\right)$$

We see that the α derivative of $\log\{\tau(\alpha)\,[1-\Phi\,(z(\alpha))]\}$ is

$$\frac{\tau'(\alpha)}{\tau}+\frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))}\left(\frac{\phi(z(\alpha))}{1-\Phi(z(\alpha))}-z(\alpha)\right)\frac{\alpha}{1-\alpha^2}. \tag{57}$$

Let $c\downarrow 0$. Then $\tau'(\alpha)\to 0$ because §D.2 shows that the right side of (44) vanishes. Next, $\zeta(\alpha,c)\to\infty$ and $z(\alpha)\to\infty$. In the limit, the second term of (57) tends to $\alpha/(1-\alpha^2)>0$, by l'Hopital's rule (61 in Online Appendix III). Thus, $\tau(\alpha)[1–\Phi\,(z(\alpha))]$ is strictly rising in α, for small $c>0$. Altogether, (54) rises in α, for small $c>0$. □

Proof of Corollary 2: To see that the number of options N and accuracy α are complements, consider the derivative $\partial\mathcal{V}/\partial\alpha$ in (54). As N rises, the r.v. that has density $\pi(\alpha,x)^{N-1}\phi(x)$ increases in the first-order stochastic sense because $\pi(\alpha,x)$ increases in x by Claim 11. Also, search duration τ rises in N, by Theorem 9. So $\partial^2\mathcal{V}/\partial\alpha\partial N\geq 0$. The claim $\partial^2\mathcal{V}/\partial N\partial c\leq 0$ follows directly from the Envelope result that $\partial\mathcal{V}/\partial c=-\tau$ (Online Appendix §I) and Theorem 9 which states τ rises in N. □

E More Search Options Proofs

E.1 Equivalent Thin Tail Characterizations

By log-concavity, $\ell = \lim_{\chi \to F^{-1}(1)} f(\chi)/[1-F(\chi)]$ exists. If F has a thin tail, $\ell = \infty$.

Claim 13 *If $F^{-1}(1)=\infty$, then $\lim_{\chi \to F^{-1}(1)} f(s+\chi)/f(\chi) = e^{-s\ell}$ for all $s > 0$.*

Proof: As $F^{-1}(1) = \infty$, for all $s>0$:

$$\lim_{\chi\to\infty} \log\left(\frac{1-F(s+\chi)}{1-F(\chi)}\right) = \lim_{\chi\to\infty} \int_0^s \frac{-f(r+\chi)}{1-F(r+\chi)} dr = \int_0^s \lim_{\chi\to\infty} \frac{-f(r+\chi)}{1-F(r+\chi)} dr = -s\ell,$$

exchanging integration and limits by the Monotone Convergence Theorem: $f/(1-F)$ is monotone if f is log-concave. For all $s > 0$, by l'Hôpital's rule and exponentiation:

$$\lim_{\chi\to\infty} \frac{f(s+\chi)}{f(\chi)} = \lim_{\chi\to\infty} \exp\left[\log\left(\frac{1-F(s+\chi)}{1-F(\chi)}\right)\right] = \exp\left[\lim_{\chi\to\infty} \log\left(\frac{1-F(s+\chi)}{1-F(\chi)}\right)\right] = e^{-s\ell}$$

Claim 14 *If $F^{-1}(1) = \infty$, F has a thin tail iff $\lim_{\chi \to F^{-1}(1)} \frac{f(s+\chi)}{f(\chi)} = 0$ for all $s>0$.*

Proof: Given a thin tail, $\ell = \infty$ and $f(s+\chi)/f(\chi) \to 0$ for $s > 0$ by Claim 13. But if $\lim_{\chi\to\infty} f(s+\chi)/f(\chi) = 0\ \forall s > 0$, then $\lim_{\chi \to F^{-1}(1)} f(\chi)/[1-F(\chi)]$ equals

$$\lim_{\chi\to\infty}\left(\int_0^\infty \frac{f(s+\chi)}{f(\chi)} ds\right)^{-1} = \left(\lim_{\chi\to\infty}\int_0^\infty \frac{f(s+\chi)}{f(\chi)} ds\right)^{-1} = \left(\int_0^\infty \lim_{\chi\to\infty} \frac{f(s+\chi)}{f(\chi)} ds\right)^{-1} = \infty.$$

by continuity and the Monotone Convergence Theorem. Hence, F has a thin tail. □

E.2 Increasing Number of Options: Proofs of Theorems 9–10

Index the striking, recall and quitting hazard rates by the number of options N. We argue that $\mathcal{K}_n^N$, $\mathcal{R}_n^N$, and $\mathcal{Q}_n^N$ weakly fall in N, and so limits $\mathcal{K}_n^\infty$, $\mathcal{R}_n^\infty$, and $\mathcal{Q}_n^\infty$ exist.

Claim 15 (Known Factors) *Conditional on hitting stage n, $\mathcal{X}_n^N$ converges to $G^{-1}(1)$ in probability as $N \to \infty$, i.e., $\lim_{N\to\infty} P(\mathcal{X}_n^N \le a|\text{enter stage } n) = 0$ if $a < G^{-1}(1)$.*

Proof: By (20) and (21), the cdf of $\mathcal{X}_n$ is

$$P(\mathcal{X}_n^N \le a|\text{enter stage } n) = \frac{\int_{u-\zeta(c)}^{a} \delta(\chi,c)^{n-1}[G(\chi)/G(a)]^{N-n} g(\chi) d\chi}{\int_{u-\zeta(c)}^{\infty} \delta(\chi,c)^{n-1}[G(\chi)/G(a)]^{N-n} g(\chi) d\chi}.$$

As $N \to \infty$, the numerator vanishes as $G(\chi)/G(a) < 1$ for all $\chi \in [u-\zeta(c), a)$, and the denominator explodes, as $G(\chi)/G(a) > 1$ for $\chi \in (a, \infty)$ and the density $g(\chi)$ has positive mass on (a, ∞) if $a < G^{-1}(1)$. Thus, $\lim_{N\to\infty} P(\mathcal{X}_n^N \le a) = 0\ \forall a < G^{-1}(1)$. $\square$

Claim 16 (Striking Chance) *Fixing n, the conditional striking chance $\mathcal{K}_n^N$ falls in N. The limit $\mathcal{K}_n^\infty = 1 - H(\zeta(c))$ if G has a thin tail, and $\mathcal{K}_n^\infty > 1 - H(\zeta(c))$ if not.*

Proof: Write $\mathcal{K}_n^N = 1 - H(\zeta(c)) + E_{\mathcal{X}_n^N}[\Gamma(s, \mathcal{X}_n^N)]$, where $\Gamma(s, \mathcal{X}_n^N)$ is Step 2's bracketed term. As $\iota(s, \chi, \zeta(c)) \equiv \int_s^\infty H(\zeta(c) - t) g(t+\chi) dt$ is log-submodular in (s, χ) by (38) and log-concavity of g, and $g(s+\chi)/g(\chi)$ weakly falls in χ by log-concavity, ($\diamondsuit$) holds: $\Gamma(s, \mathcal{X}_n^N)$ *falls in* $\mathcal{X}_n^N$. As $\mathcal{X}_n^N$ stochastically rises in N by Claim 1, $\mathcal{K}_n^N$ falls in N.

The n^{th} known factor $\mathcal{X}_n^N \to G^{-1}(1) = \infty$ in probability by Lemma 15, as $N \to \infty$. If G has a thin tail, then $\lim_{\chi\to\infty} g(s+\chi)/g(\chi) = 0$ for $s > 0$, by Claim 14, and so $g(s + \mathcal{X}_n)/g(\mathcal{X}_n^N) \downarrow 0$ as $N \to \infty$. In this case, Step 2 implies $\lim_{N\to\infty} \mathcal{K}_n^N = 1 - H(\zeta(c))$.

Assume no thin tail of G. As $\lim_{\chi\to\infty} g(t+\chi)/g(\chi) = e^{-t\ell}$ for $\ell > 0$, by Claim 13,

$$\Gamma(s, \chi) \equiv \frac{\int_s^\infty H(\zeta(c) - t) g(t+\chi)/g(\chi) dt}{\int_0^\infty H(\zeta(c) - t) g(t+\chi)/g(\chi) dt} \to \frac{\int_s^\infty H(\zeta(c) - t) e^{-\ell t} dt}{\int_0^\infty H(\zeta(c) - t) e^{-\ell t} dt} > 0 \text{ as } \chi \to \infty. \tag{58}$$

By the above claim ($\diamondsuit$), $\Gamma(s, \chi)$ falls in χ, tending to $\lim_{\chi\to\infty} \Gamma(s, \chi) > 0$ by (58). Since $\mathcal{X}_n^N$ increases stochastically in N, by Claim 1, $\lim_{N\to\infty} E_{\mathcal{X}_n^N}[\Gamma(s, \mathcal{X}_n^N)] > 0$ by the Continuous Mapping Theorem, and therefore, $\mathcal{K}_n^\infty > 1 - H(\zeta(c))$. $\square$

Claim 17 (Recall) *$\mathcal{R}_n^N$ falls in N and $\mathcal{R}_n^\infty = 0$ iff G has a thin tail. The limit $\mathcal{E}_n^\infty \equiv \mathcal{R}_n^\infty + \mathcal{K}_n^\infty$ is $1 - H(\zeta(c))$ if G has a thin tail, and $\mathcal{E}_n^\infty \in (1 - H(\zeta(c)), 1)$ if not.*

Proof: As $\mathcal{X}_n$ stochastically rises in N (Claim 1), and $B(\chi, n)$ falls in χ (Step 4), $\mathcal{R}_n^N = E_{\mathcal{X}_n}[B(\mathcal{X}_n, n)]$ falls in N. So $\mathcal{R}_n^\infty = \lim_{\chi\to G^{-1}(1)} B(\chi, n)$ by the Continuous Mapping Theorem, as $\mathcal{X}_n \to G^{-1}(1)$ in probability if $N \to \infty$ (Claim 15).

If G has a thin tail, and $s > 0$, then $g(s+\chi)/g(\chi) \downarrow 0$ as $\chi \to G^{-1}(1) = \infty$, by Claim 14; so $B(\chi, n) \downarrow 0$ by (32). With no thin tail, $(1 – \Gamma(s, \chi)^{n-1})$ in (32) is boundedly positive as $\chi \to G^{-1}(1)$ (by (58)). So $\lim_{\chi\to G^{-1}(1)} B(\chi, n) > 0$ by (32), i.e. $\mathcal{R}_n^\infty = 0$ iff G has a thin tail.

By Claim 1, $\mathcal{E}_n^\infty = 1 - H(\zeta(c))$ if G has a thin tail. Else, $\mathcal{E}_n^\infty \in (1 - H(\zeta(c)), 1)$. $\square$

Claim 18 (Quitting) *For any G, $\mathcal{Q}_n^N$ falls in N, and tends to the limit $\mathcal{Q}_n^\infty = 0$.*

Proof: Expanding $\mathcal{Q}_n \equiv q_n/\sigma_n$ using (24) and (7), respectively:

$$\mathcal{Q}_n^N = \frac{\delta(u-\zeta(c),c)^n G(u-\zeta(c))^{N-n}}{n\int_{u-\zeta(c)}^{\infty}\delta(\chi,c)^{n-1}G(\chi)^{N-n}g(\chi)d\chi}.$$

Easily, $\mathcal{Q}_n^N$ falls in N, since $G(\chi)/G(u-\zeta(c)) > 1$ except at $\chi = u-\zeta(c)$, and thus $[G(\chi)/G(u-\zeta(c))]^{N-n}$ is monotone in N. By the monotone convergence theorem, we can swap the (infinite) limit as $N\to\infty$ and integration: $\lim_{N\to\infty}\mathcal{Q}_n^N = \mathcal{Q}_n^\infty = 0$. □

Claim 19 (Duration) *Search duration rises in N.*

Proof: Since $\mathcal{Q}_n$, $\mathcal{R}_n$ and $\mathcal{K}_n$ fall in N, search duration τ rises in N. For the striking hazard rate $\mathcal{S}_k \equiv 1-\sigma_{k+1}/\sigma_k$ yields (by a telescoping product) the survival chance formula $\sigma_k = \sigma_1\Pi_{j=1}^{k-1}(1-\mathcal{S}_j)$. As N rises, so does this product: $\sigma_1 = P(\mathcal{X}_1 > u-\zeta(c))$ rises by Claim 1, and every $\mathcal{S}_j \equiv \mathcal{Q}_j + \mathcal{E}_j$ falls. Duration $\tau \equiv \sum_{k=1}^N \sigma_k$ rises in N. □

Claim 20 (Limit Recall Chance) *Absent a thin tail, $\mathcal{R}_n^\infty$ rises in dispersion of $\mathcal{X}$.*

Proof: By (36) and $\nu(s,\chi,\zeta(c)) \equiv \int_s^\infty H(\zeta(c)-t)g(t+\chi)dt/\int_0^\infty H(\zeta(c)-t)g(t+\chi)dt$,

$$\frac{B(\chi,n)}{n-1} = \int_0^\infty \frac{H(\zeta(c)-s)g(s+\chi)}{g(\chi)}\left[\frac{\int_s^\infty h(\zeta(c)-t)\frac{g(t+\chi)}{g(\chi)}dt}{\int_s^\infty H(\zeta(c)-t)\frac{g(t+\chi)}{g(\chi)}dt}\right]\left[\frac{\int_s^\infty H(\zeta(c)-t)\frac{g(t+\chi)}{g(\chi)}dt}{\int_0^\infty H(\zeta(c)-t)\frac{g(t+\chi)}{g(\chi)}dt}\right]^{n-1} ds.$$

By Claim 13, with no thin tail, $\lim_{\chi\to G^{-1}(1)} g(s+\chi)/g(\chi) = e^{-s\ell}$ for all $s>0$, where $\ell \equiv \lim_{a\to 1} g(G^{-1}(a))/(1-a)$. Since $\mathcal{R}_n^\infty \equiv \lim_{N\to\infty} E_{\mathcal{X}_n^N}[B(\mathcal{X}_n^N,n)] = \lim_{\chi\to G^{-1}(1)} B(\chi,n)$:

$$\mathcal{R}_n^\infty = (n-1)\int_0^\infty H(\zeta(c)-s)e^{-s\ell}\left[\frac{\int_s^\infty h(\zeta(c)-t)\,e^{-t\ell}dt}{\int_s^\infty H(\zeta(c)-t)e^{-t\ell}dt}\right]\left[\frac{\int_s^\infty H(\zeta(c)-t)e^{-t\ell}dt}{\int_0^\infty H(\zeta(c)-t)e^{-t\ell}dt}\right]^{n-1} ds.$$

The limit $\ell \equiv \lim_{a\to 1} g(G^{-1}(a))/(1-a)$ falls in the dispersion of $\mathcal{X}$. As in the proof of Step 4, $\int_s^\infty H(\zeta(c)-t)e^{-t\ell}dt$ is log-submodular in (ℓ,ζ) and in (ℓ,s), by log-concavity of H. So $\mathcal{R}_n^\infty$ rises in the dispersion of $\mathcal{X}$, as each bracketed factor above falls in ℓ. □

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Proposed Online Appendix

I More Value Function Differentiability

By Lemma 4, the slope $V_n'(\Omega)$ in (4) is the chance that the best-so-far Ω will be eventually exercised. In the same spirit, now we show that the derivative of $-V_n(\Omega)$ with respect to the search cost c equals the expected number of remaining searches.

Claim 21 (Differentiability) *The period n value function $V_n(\Omega_n)$ is differentiable in the cost c when the stage n best option so far $\Omega_n \neq \bar{w}_{j+1}$ for $j \in \{n+1, \ldots, N\}$. The derivative $-\partial V_n(\Omega_n)/\partial c$ is the expected number of remaining searches.*

PROOF: Assume $\Omega_n \neq \bar{w}_{j+1} = \chi_{j+1} + \zeta(c)$ for $j \in \{i, \ldots, N\}$. The terminal value function is $V_N(\Omega_N) = \Omega_N$. Since $\partial V_N(\Omega_N)/\partial c = 0$, all claims are true at stage N. Suppose the statements hold at stage $n+1$. At stage n, if $\Omega_n > \bar{w}_{n+1}$ then Sam stops searching. By (4), we have $V_n(\Omega_n) = \Omega_n$ on $[\bar{w}_{n+1}, \infty)$ and so $\partial V_n(\Omega_n)/\partial c = 0$. If $\Omega_n < \bar{w}_{n+1}$, then Sam continues to stage $n+1$. In this case, $-\partial V_n(\Omega_n)/\partial c = 1 - [\partial V_{n+1}(\Omega_n)/\partial c]F_{n+1}(\Omega_n) - \int_{\Omega_n}^{\infty}[\partial V_{n+1}(z)/\partial c]dF_{n+1}(z)$ by (4). The integral exists since $\partial V_{n+1}(z)/\partial c$ exists except at finitely many points. Then $-\partial V_n(\Omega_n)/\partial c$ equals one plus the expected number of remaining searches. This proves the induction. □

We now show that the value $\mathcal{V}$ is differentiable in u and c as long as $\mathcal{X}$ is non-degenerate, namely

$$\frac{\partial \mathcal{V}(u,c)}{\partial u} = q \quad \text{and} \quad \frac{\partial \mathcal{V}(u,c)}{\partial c} = -\tau. \tag{59}$$

Consider the stage after the realization of $\vec{\chi} \equiv \{\chi_1, \chi_2, \ldots, \chi_N\}$ but before Sam explores any option. Let $V_0(\vec{x})$ be Sam's expected payoff. By Lemma 4 and Claim 21, $V_0(\vec{\chi})$ is differentiable in u and c except when $u = \chi_j + \zeta(c)$ for any $j \in \{1, \ldots, N\}$. Sam's ex ante payoff before the known factors are realized is $\mathcal{V}(u,c) = E[V_0(\vec{\mathcal{X}})]$ where the expectation is taken over $\vec{\mathcal{X}}$. Both $\partial \mathcal{V}(u,c)/\partial u$ and $\partial \mathcal{V}(u,c)/\partial c$ exist because $\partial V_0(\vec{\mathcal{X}})/\partial u$ and $\partial V_0(\vec{\mathcal{X}})/\partial c$ exist except on a set of measure zero.[40]

The slope $\partial V_0(\vec{\chi})/\partial u$ is the quitting chance given $\vec{\chi}$, by Lemma 4. And the expected search time given $\vec{\chi}$ is $-\partial V_0(\vec{\chi})/\partial c$, by Claim 21. So (59) holds. □

[40] By the Dominated Convergence Theorem (DCT), if $\partial V_0(\vec{\mathcal{X}})/\partial c$ exists except on a measure zero set and $|\partial V_0(\vec{\mathcal{X}})/\partial c|$ is bounded above by a constant a.s. for all c, then $\partial \mathcal{V}(u,c)/\partial c = \partial E[V_0(\vec{\mathcal{X}})]/\partial c = E[\partial V_0(\vec{\mathcal{X}})/\partial c]$. Here the slope of $V_0(\vec{\mathcal{X}})$ with respect to c is bounded in $[-N, 0]$, as Sam at most searches N times. Hence the DCT applies. A similar proof works for $\partial \mathcal{V}(u,c)/\partial u$.

II Omitted Algebra: Proof of Equation (48)

Put $u = \zeta(\alpha, c)$ and $x = \ell(\alpha, u, c)$ in (42). Then $(\partial/\partial\alpha)\pi(\alpha, \ell(\alpha, u, c))$ equals

$$\begin{aligned}
& \int_{\ell(\alpha,u,c)}^{\infty} \phi\left(\frac{u-\alpha s}{\sqrt{1-\alpha^2}}\right)\frac{(\alpha u - s)}{\sqrt{1-\alpha^2}} d\Phi(s) + \frac{\partial \ell(\alpha,u,c)}{\partial \alpha}\phi(\ell(\alpha,u,c))\left[1-\Phi\left(\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}\right)\right] \\
= & \ \phi(u)\int_{\ell(\alpha,u,c)}^{\infty} \phi\left(\frac{s-\alpha u}{\sqrt{1-\alpha^2}}\right)\frac{(\alpha u - s)}{\sqrt{1-\alpha^2}} ds + \frac{\partial \ell(\alpha,u,c)}{\partial \alpha}\phi(\ell(\alpha,u,c))\left[1-\Phi\left(\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}\right)\right] \\
= & -\frac{\phi(u)}{\sqrt{1-\alpha^2}}\phi\left(\frac{\ell(\alpha,u,c)-u\alpha}{\sqrt{1-\alpha^2}}\right) + \frac{\partial \ell(\alpha,u,c)}{\partial \alpha}\phi(\ell(\alpha,u,c))\left[1-\Phi\left(\frac{\zeta(\alpha,c)}{\sqrt{1-\alpha^2}}\right)\right] \\
= & \ \phi(\ell(\alpha,u,c))\phi(\ell(\alpha,u,c))\left[1-\Phi\left(\zeta(\alpha,c)/\sqrt{1-\alpha^2}\right)\right]
\end{aligned}$$

The second equality uses the Gaussian density property $\partial\phi(s)/\partial s = -s\phi(s)$.

III Omitted Algebra for Proof of Theorem 8

A Key Inequality. We verify $\pi(0,0) = 1/2(1 + \Phi(\zeta(0,c))) > \Phi(\zeta(0,c)+c)$, or

$$\frac{1-\Phi(\zeta(0,c)+c)}{1-\Phi(\zeta(0,c))} \geq \frac{1}{2}. \tag{60}$$

As $c \uparrow \infty$, $\zeta(0,c) \to -\infty$ by (13). By integration by parts of (13) yields $\zeta(0,c) + c = E[\max\{Z, \zeta(0,c)\}] \to E[Z] = 0$ as $c \uparrow \infty$, the LHS of (60) has limit 1/2 as $c \to \infty$.

Log-differentiate (60) in c, using $\partial\zeta(0,c)/\partial c = 1/[1-\Phi(\zeta(0,c))]$ from (13):

$$\begin{aligned}
& \frac{\phi(\zeta(0,c)+c)}{1-\Phi(\zeta(0,c)+c)}\frac{\Phi(\zeta(0,c))}{1-\Phi(\zeta(0,c))} - \frac{\phi(\zeta(0,c))}{1-\Phi(\zeta(0,c))}\frac{1}{1-\Phi(\zeta(0,c))} \\
\leq & \left(\frac{\phi(\zeta(0,c))}{1-\Phi(\zeta(0,c))} + c\right)\frac{\Phi(\zeta(0,c))}{1-\Phi(\zeta(0,c))} - \frac{\phi(\zeta(0,c))}{1-\Phi(\zeta(0,c))}\frac{1}{1-\Phi(\zeta(0,c))}
\end{aligned}$$

The first inequality owes to $\phi(\zeta+c)/[1-\Phi(\zeta+c)] = E[Z|Z > \zeta+c]$, where $Z \sim N(0,1)$, and $E[Z|Z > \zeta + c] \leq E[Z|Z > \zeta] + c$ by Heckman and Honore (1990). This has the sign of

$$c\Phi(\zeta(0,c)) - \phi(\zeta(0,c)) = -[1-\Phi(\zeta(0,c))](c + \zeta(0,c)) < 0.$$

Equality follows by integrating (13) by parts at $\alpha = 0$, to get $\zeta(1-\Phi(\zeta)) - \phi(\zeta) = c$. The last inequality reflects $\zeta(0,c)$ strictly falling in α and $\zeta(1,c) = -c$, by (13).

Altogther, the LHS of (60) strictly falls in c, and inequality holds at all $c > 0$. □

A Key Limit. We verify that

$$\lim_{z\to\infty}\frac{\phi(z)}{1-\Phi(z)}\left(\frac{\phi(z)}{1-\Phi(z)}-z\right)\to 1. \tag{61}$$

Now, $\int_z^\infty s\phi(s)ds = -\int_z^\infty d\phi(s) = \phi(z)$, as a Gaussian density ϕ obeys $\phi'(s) = -s\phi(s)$. Then

$$\frac{\phi(z)}{1-\Phi(z)} = \frac{\int_z^\infty s d\Phi(s)}{1-\Phi(z)} = \frac{\int_z^\infty [1-\Phi(s)]ds}{1-\Phi(z)} + z.$$

integrating by parts. Hence,

$$\lim_{z\to\infty}\frac{\phi(z)}{1-\Phi(z)}\left(\frac{\phi(z)}{1-\Phi(z)}-z\right) = \lim_{z\to\infty}\frac{\phi(z)}{1-\Phi(z)}\frac{\int_z^\infty [1-\Phi(x)]dx}{1-\Phi(z)} = \lim_{z\to\infty}\frac{\phi(z)}{1-\Phi(z)}\frac{1-\Phi(z)}{\phi(z)}$$

This limit is one, where the second equality uses L'Hôpital's rule. □

IV Mean Accepted Option and Quit Payoff

We prove the claim in footnote 32 in §6 that a higher quit payoff worsens the accepted eventual option, even thought one might accept that higher quit payoff.

Claim 22 (Mean Accepted Option) *$\frac{\partial}{\partial u}E[\mathcal{W}^A(\alpha, c)] < 0$ for large enough N*

Proof: Recalling (18), the optimal payoff is $\mathcal{V} = uq + E[\mathcal{W}^A(\alpha, c)](1-q) - \tau c$, and so $\partial\mathcal{V}/\partial u = q$, by the Envelope Theorem (justified in Online Appendix I). Since $E[\mathcal{W}^A(\alpha, c)] = \mathcal{V}(\alpha, c) + c\tau(\alpha, c)$ from (15), we have:

$$\begin{aligned}\frac{\partial}{\partial u}E[\mathcal{W}^A(\alpha, c)] &= q - \frac{cN}{\alpha}\pi(\alpha, \ell(\alpha, u, c))^{N-1}\phi(\ell(\alpha, u, c)) \\ &= \pi(\alpha, \ell(\alpha, u, c))^{N-1}\left[\pi(\alpha, \ell(\alpha, u, c)) - \frac{cN}{\alpha}\phi(\ell(\alpha, u, c))\right].\end{aligned}$$

The first line uses (43) and $\ell_u(\alpha, u, c) = 1/\alpha$ from (14). The second line owes to $q = \pi(\alpha, \ell(\alpha, u, c))^N$ from (47). The right side is negative for large N. □