The Economics of Counterfeiting

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Currency Counterfeiting: Past and Present

- Counterfeiting has forever been a thorn in the flesh of fiat money
- In the Civil War, counterfeiting helped push the Confederate currency out of use
- U.S. passed rate is now about 1-2 per 10,000 notes
- In 2007, the direct cost to USA public of passed money was a record \$61*M*
- Other costs are surely much larger:
  - undermining faith in the U.S. currency
  - driving notes out of circulation (“No \$100 bills allowed”)
  - eliminating market transactions

Document Counterfeiting

- counterfeit checks are a much larger problem — estimated $20 billion in 2003 (“Nigerian scams”)
- “Record $6 Trillion of Fake U.S. Bonds Seized” (February 17, 2012)

The U.S. embassy in Rome has examined the securities dated 1934, which had a nominal value of $1 billion apiece, they said in the statement. “Thanks to Italian authorities for the seizure of fictitious bonds for $6 trillion” the embassy said.

Goods Counterfeiting

- “The Crime of the 21st Century” (FBI): fake, fake, fake
- In 2000, trade in counterfeit goods was $450 billion
- goods counterfeiting is critically different: price is a market quantity, and people buy them for themselves as final consumers
- gray line exists between counterfeits and “knock-offs” (fake Prada), bought because they are cheaper

What has Been Said on Counterfeiting

- The existing literature on counterfeit money consists of matching models that explore the possibility of monetary models with circulating counterfeit fiat money
- People may see random signals of counterfeit status
- In this “general equilibrium” literature, people to discount money due to counterfeiting

What has Been Said on Counterfeiting

- How realistic is this?
- Discounting happens for goods, but not much for notes and documents, except possibly when counterfeiting runs amok (Civil War)
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- ♢ Big picture: A counterfeiting game by bad guys knowingly trying to pass bad money induces a collateral passing game by good guys trying not to unknowingly accept bad money
- ♠ Let us examine these games in reverse order

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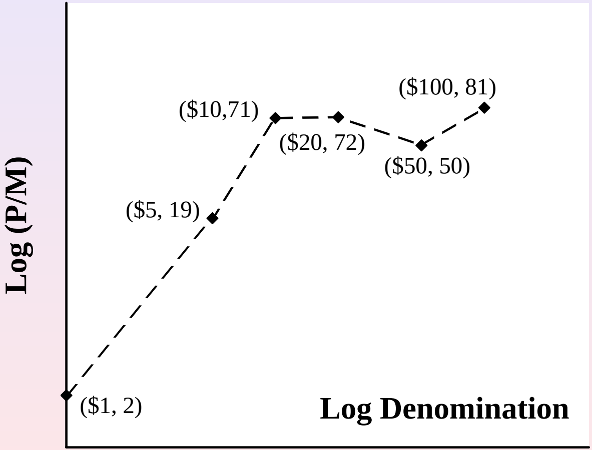
1. "Hot Potato Game" (among good guys passing money)
2. Counterfeiters Model 1: Non-optimizing Criminals
   ⇢ cannot explain data
3. Counterfeiters Model 2: Criminals Optimize Quantity
   ⇢ partially explain data (at low denominations)
4. Counterfeiters Model 3: Criminals Optimize Quality
   ⇢ explains data

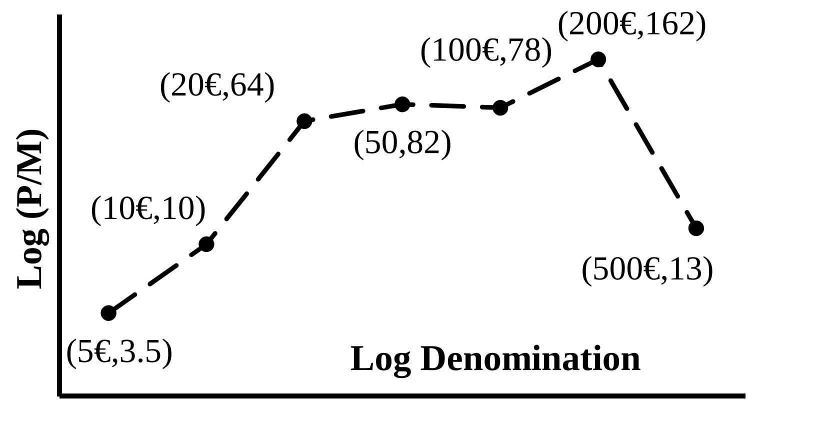
↬ Model 3 = "Cat and Mouse Game"

Seized and Passed Counterfeit Money

- *Seized counterfeit money* are bad notes taken from "bad guys"
- *Passed counterfeit money* are bad notes found in the possession of "good guys"
- Knowingly trying to pass counterfeit money is a crime

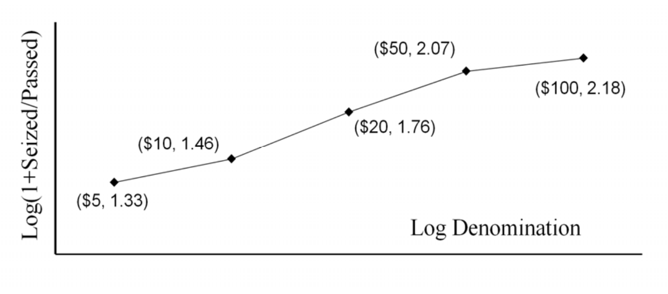
USA Passed Counterfeit Money Rate, 1995–2007





Log (P/M)
(5€,3.5)
(10€,10)
(20€,64)
(50,82)
(100€,78)
(200€,162)
(500€,13)
Log Denomination

The Counterfeit-Passed Ratio (1995-2007)



Our New Decision Margins

- We begin with a new decision margin for "good guys": How carefully do I examine the notes I acquire?
- This costly verification assumption alone explains the rising passing fraction in the note value.
- Where our story fails at the high denominations, we show that this owes to our assumption that the quality of counterfeits is constant

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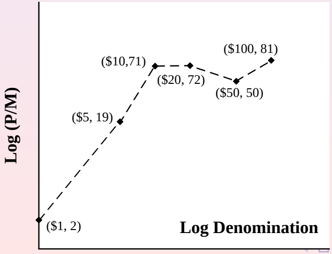
- Where our story fails at the high denominations, we show that this owes to our assumption that the quality of counterfeits is constant
- One missing additional new decision margin does the trick — an endogenous quality choice by counterfeiters
- We produce a tractable framework for answering many empirical questions, and apply it where we have data
- That the theory even works is amazing, for we argue that the costs are astoundingly low — at most $1/4$ cent to check a \$100 note, much less for other notes!

Can We Explain Passed Counterfeit Money?

- First, let's ignore incentives of bad guys $\Rightarrow$ exogenous counterfeiting inflow is a fraction m of money supply
- $\kappa =$ counterfeiting rate, $v =$ verification rate
- $\pi =$ passed rate
- counterfeit outflow (passed money) = fixed inflow:

$$\pi = \kappa v = m \qquad \text{(steady-state)}$$

- This model fails to explain the passed data, except by assuming that entry so happens to generate it



Costly Vigilance: What Good Guys Do

- Vigilance = action undertaken by good guys
- Verifying authenticity of money is costly and stochastic
- Catching a counterfeit note with chance $v \in (0, 1)$ mentally costs $\chi(v)$

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Properties

- $\chi(v)$ is a smooth increasing and strictly convex function
- We must assume log-concavity: $[\log(\chi(v))]'' \leq 0$
- Example: $\chi(v) = v^B$ with $B \geq 2$.

The Hot-Potato Passing Game

The Hot-Potato Passing Game

- *Large Game:* A continuum of good guys randomly match each period, trading a denomination $\Delta > 0$.
- $\kappa \in [0, 1]$ is the endogenous fraction of counterfeit notes
- $v \in [0, 1]$ is the endogenous average verification rate
- In the *hot-potato passing game*, individuals choose $\hat{v}$ to minimize

$$\kappa(1 - \hat{v})v\Delta + \chi(\hat{v})$$

- For a given counterfeit rate κ, this is a supermodular verification game $\Rightarrow$ increasing best reply function
- Indeed, the FOC yields

$$\kappa v\Delta = \chi'(v^*)$$

- Everyone faces the same decision problem $\Rightarrow v^* = v$

- The symmetric Nash equilibrium counterfeiting rate in the large game is

$$\kappa = \frac{\chi'(v)}{v\Delta}$$

- One could have allowed heterogeneity here among verifiers, if needed.
- The passed rate $\pi = \kappa v$ is therefore

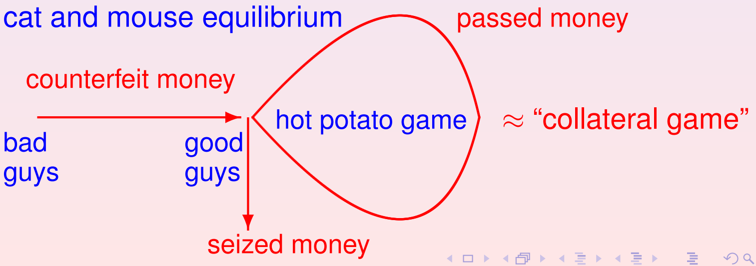
$$\pi = \frac{\chi'(v)}{\Delta} = \frac{\text{marginal verification cost}}{\text{denomination}}$$

A Big Picture: Police, Good Guys, & Bad Guys

- We cheaply introduce police into the story: When innocents verify with chance v, the fraction crooks pass into circulation is the passing fraction $f(v) \leq 1 - v$

$\Rightarrow$ $f(1) = 0$ while $f(0) > 0$ with some police enforcement

- $f' < 0$ and diminishing returns $f'' > 0$. Also, $f'(0) > -\infty$
- We will also assume log-concavity of f
- Example: If police and verifiers independently find fractions γ and v of money, then $f(v) = (1 - \gamma)(1 - v)$



Bad Guys

"I'm a local craftsperson—I make money."

Trivial but Optimizing Model of Counterfeiters

- Counterfeiting inflow comes from a competitive market
- Criminals produce an expected quantity x of counterfeit Δ notes at cost $c(x)$, smooth, increasing, and convex
- They are eventually caught, & pay a legal penalty $L > 0$
- A fraction $f(v)$ of notes pass into circulation
- Profits are then revenues less physical and legal costs:

$$\Pi(x, v) = xf(v)\Delta - c(x) - L$$

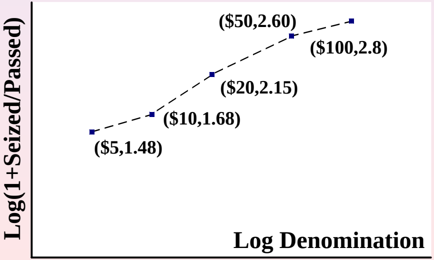
- Optimal quantity $\Rightarrow f(v)\Delta = c'(x)$
- Crime does not pay:
- Free entry $\Rightarrow$ zero profits $\Pi(x, v) = 0$

$$\Rightarrow \text{producer surplus} = x^* c'(x^*) - c(x^*) = L$$

- Quantity x^* is independent of Δ!!

Trivial but Optimizing Model of Counterfeiters

- Our optimal quantity relation implies that the passing fraction is inverse to the note $f(v) = c'(x^*)/\Delta$
- Good: Near the positive least counterfeit note, v vanishes, and so does the passed rate $\pi = \chi'(v)/\Delta$
- Bad? The passed rate should vanish as $\Delta \uparrow \infty$
- Bad: The (inverse) counterfeit-passed ratio should double when the denomination doubles. It does not.



Our Model: Variable Quality Counterfeiting

- We henceforth fix the quantity x, and assume instead that counterfeit quality is a choice variable.
- Quality inflates the passing rate
- The $100 note is high quality, and passes more easily
- Supernote, Columbian counterfeits of $50 & $100 bills, vs. “Counterfeit Millionaire” vs. youthful counterfeits

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- Supernote, Columbian counterfeits of $50 & $100 bills, vs. "Counterfeit Millionaire" vs. youthful counterfeits
- Quality q note scales verification costs to $e = q\chi(v)$
- Critically, quality has a cardinal meaning!
- Quality q (of quantity x notes) incurs cost $c(q)$, a smooth, increasing, and strictly convex function
- Good guys choose vigilance effort e not observing q
- Notice that the verification intensity is the endogenous outcome of good guys' efforts and bad guys' quality:

$$v = V(e, q) \qquad \Leftrightarrow \qquad e = q\chi(v)$$

The Verification Function

- To see how effort and quality interact to fix a verification rate, differentiate the identity $q\chi(V(e,q)) \equiv e$:

$$q\chi' V_q + \chi = 0 \Rightarrow \boxed{V_q = -\frac{\chi}{q\chi'} < 0}$$

$$q\chi' V_e \equiv 1 \Rightarrow \boxed{V_e = \frac{1}{q\chi'} > 0}$$

$$\boxed{q^2 V_{qq} = \frac{\chi}{\chi'} + \left(\frac{\chi}{\chi'}\right)^2 \left(\frac{\chi'}{\chi} - \frac{\chi''}{\chi'}\right)}$$

- There are diminishing returns to quality, or $V_{qq} > 0$, if costs χ are log-concave (thus assumed from now on)
- $\chi(v) = v^\gamma$ is convex and log-concave for all $\gamma > 1$
- $\chi(v) = e^{\gamma v}$ is convex and log-linear for all $\gamma > 0$

The Supply of Counterfeit Money

- Counterfeiting inflow comes from a competitive market
- Criminals produce a quality q of counterfeit Δ notes at cost $c(q)$, smooth, increasing, and convex
- Let us fix the quantity x. Profits are thus:

$$\Pi(q,e) = f(V(e,q))x\Delta - c(q) - L$$

- Free entry $\Rightarrow$ zero profits

$$xf(V(e,q))\Delta = c(q) + L$$

- Optimal quality

$$xf'(V(e,q))V_q(e,q)\Delta = c'(q)$$

Cat and Mouse Equilibrium

- A *cat and mouse equilibrium* is a pair (q, e) yielding zero profits, for which the quality q is profit-maximizing.
- Both effort and quality adjust as counterfeiting evolves

Cat and Mouse Equilibrium

Theorem (Non-Existence)

No cat and mouse equilibrium exists for notes $\Delta \leq \underline{\Delta}$.

- For if $\Delta < \underline{\Delta}$, then profits are less than $\underline{\Delta} x f(0) - L = 0$.
- If $\Delta = \underline{\Delta}$, then zero profits requires that quality vanish.
- Verification would then be perfect for all effort $e > 0$, and counterfeiters would lose at least $L > 0$.

We henceforth restrict to notes $\Delta > \underline{\Delta}$.

Let's explore the battle between effort and quality, as various parameters change.

Zero profit locus $\bar{\Pi}$

$$\Delta x f(v) = c(q) + L$$

Optimal quality locus Q^*

$$-\Delta x f'(v) \frac{\chi(v)}{\chi'(v)} = qc'(q)$$

By taking logs, write these as respectively

$$\begin{aligned} F(v) + \log \Delta &= T(q) \\ G(v) + \log \Delta &= U(q) \end{aligned}$$

We assume not only strictly log-concave costs $\chi(v)$ and but also a log-concave passing function $f(v)$, so that:

$$G'(v) - F'(v) \equiv \frac{f''}{f'} - \frac{f'}{f} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} > 0$$

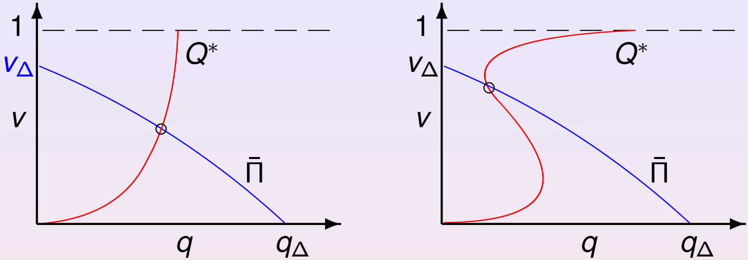


Figure : **Zero Profit and Optimal Quality Curves.** The zero profit curve $\bar{\Pi}$ monotonely slopes down from $(0, v_\Delta)$ to $(q_\Delta, 0)$, and the optimal quality locus Q^* initially rises. The left panel captures a monotone Q^* curve with no police interdiction. The right panel allows police interdiction: Any negatively-sloped portion of Q^* is steeper than the zero profit curve $\bar{\Pi}$ at an equilibrium.

When Does Optimal Quality Locus Slope Up?

Q^* slopes upward for a robust class of models with diminishing police efficacy, i.e.

$$\frac{f''}{f'} + \frac{\chi'}{\chi} - \frac{\chi''}{\chi'} > 0 \tag{1}$$

- Example: Assume $f(v) = (1 - \gamma v)(1 - v)$.
- $\gamma = 0$: no police interdiction
- $0 \leq \gamma < 1$: f is monotone decreasing, convex and log-concave, with $f(0) > 0 = f(1)$.
- With geometric verification costs $\chi(v) = v^B$, inequality (1) reduces to $vf''(v)/f'(v) \geq -1$, i.e. $\gamma \leq 1/3$

Theorem (Legal Costs)

Assume that legal costs rise. Then the verification effort and rate each fall, and the least counterfeit note $\underline{\Delta}$ rises. the counterfeit quality surely falls at low and high notes, and always falls if the optimal quality curve slopes upward.

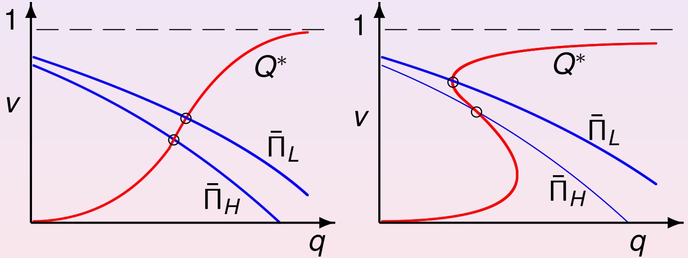


Figure : **Shifting Legal Costs**. The optimal quality locus Q^* is unaffected by legal costs. When legal costs rise, verification v falls, while quality surely falls for low and high notes, and always falls if the Q^* curve is locally rising.

Theorem (Verification Costs)

Lower verification costs raises the verification rate, raises verification effort, and lowers counterfeit quality.

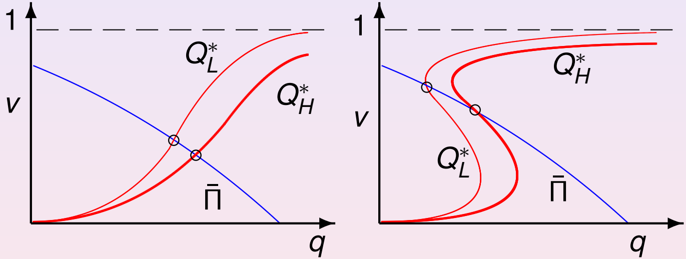
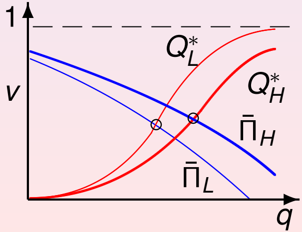


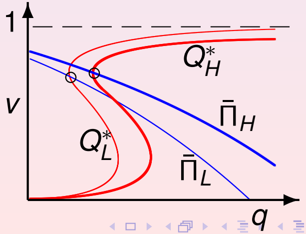
Figure : **Easier Verification.** Assume the optimal quality curve slopes up. As verification costs fall, the zero profit curve $\bar{\Pi}$ is fixed, while the optimal quality locus Q^* shifts left (from H to L, thick to thin). So verification rises, while quality falls.

Theorem (Denomination)

(*a*) *The verification effort and verification rate, and the counterfeit quality, vanish as* $\Delta \downarrow \underline{\Delta}$.
(*b*) *Effort and quality monotonically rise in the note* $\Delta > \underline{\Delta}$.
(*c*) *The verification rate increases in* Δ *at low and high* Δ*, and increases for all* $\Delta > \underline{\Delta}$ *given a monotone quality cost elasticity:*

$$\left(\frac{qc'(q)}{c(q)}\right)' \geq 0$$





- *Technological improvement* lowers the production costs of any quality: The quality $Q(q, t)$ that costs $c(q)$ given technology t obeys $Q_t < 0 < Q_q$.
- Opposite to the denomination comparative static, the optimal quality curve Q^* shifts farther right than $\bar{\Pi}$.

Theorem (Technology)

Technological improvement raises counterfeit quality, raises the verification effort but reduces the verification rate.

A Stable Multimarket Equilibrium

A *counterfeiting equilibrium* is a triple (q^*, e^*, κ^*) yielding equilibrium in each market:

1. Verifiers' effort e^* and counterfeit quality q^* are a cat and mouse equilibrium.
2. Given counterfeit quality q^*, the effort e^* by good guys is an equilibrium of the hot potato game for the counterfeiting rate $\kappa^* \in (0, 1)$, namely:

$$\kappa = \frac{q\chi'(v)}{v\Delta} = \frac{\text{marginal verification cost}}{\text{discovery rate} \times \text{denomination}}$$

A Stable Multimarket Equilibrium

1. Cat and Mouse Equilibrium *2. Hot Potato Game Equilibrium*

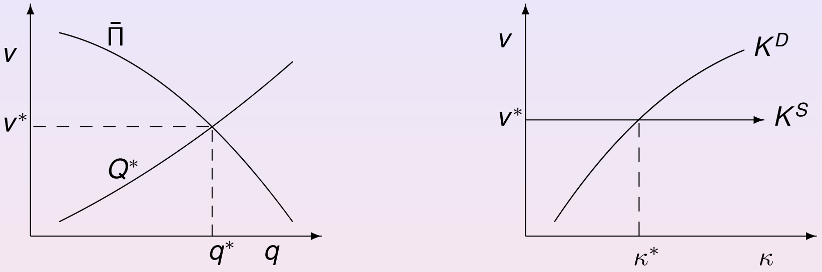


Figure : **Two Sector Equilibrium Logic.** The same verification rate must clear two counterfeit money markets — for criminals and verifiers. The cat and mouse equilibrium in (q, v)-space (left) yields the verification rate v^* — captured in the infinitely elastic derived counterfeiting supply for the hot-potato game in (κ, v)-space (right).

Side Remark on Heterogeneous Bad Guys

- Since the counterfeiting rate is a free variable, we can solve the games sequentially: first the cat and mouse game and then the hot-potato game.
- With heterogeneous criminals, this logic fails, since the marginal criminal who determines the counterfeiting rate will care about the verification rate.

An Example of a Cat and Mouse Equilibrium

- Assume geometric verification and quality production cost functions $\chi(v) = v^B$ and $c(q) = q^A$, with $A, B > 1$.
- So $\chi(v)$ and $c(q)$ are convex and χ is log-concave.
- Assume no police, so that $f(v) = 1 - v$. Then

$$\Delta x(1 - v) - q^A - L = 0 \qquad \Leftarrow \text{zero profit}$$

$$Aq^A - \Delta x v / B = 0 \qquad \Leftarrow \text{optimal quality}$$

- $v = q = 0 \Rightarrow$ least counterfeit note is $\underline{\Delta} = L/x$
- Putting $\bar{v} = AB/(1 + AB) < 1$, the solution is

$$q^A = (1 - \bar{v})(\Delta x - L) \qquad \text{and} \qquad v = \bar{v}(1 - \underline{\Delta}/\Delta) \quad (\clubsuit)$$

- Effort $e = qv^B$ rises proportionately faster than q in Δ.

Lemma (Slopes)

The constant counterfeiting locus $\bar{K}$ has a negative slope, but greater than $\bar{\Pi}$. In addition, the slope of $\bar{K}$ is less than Q^* given

$$\frac{vf''(v)}{f'(v)} + \frac{v\chi'(v)}{\chi(v)} \geq 1$$

In this case, the $\bar{K}$ locus lies between the optimal quality locus Q^* and the zero profit curve $\bar{\Pi}$.

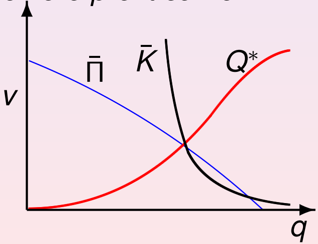


Figure : **Counterfeiting Rate.**

The Constant Counterfeiting Rate Locus

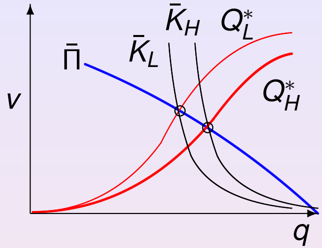


Figure : **Counterfeiting Rate.** With easier verification, the optimal quality locus shifts left (from Q_H to Q_L). This shifts to a lower constant counterfeiting rate locus $\bar{K}_L$, that is also lower *also* because the verification cost function has fallen.

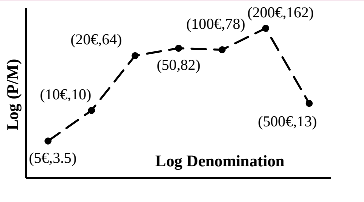
Theorem *The counterfeiting rate rises if the legal costs ℓ fall, the verification costs χ rise, or there is technological improvement in counterfeiting.*

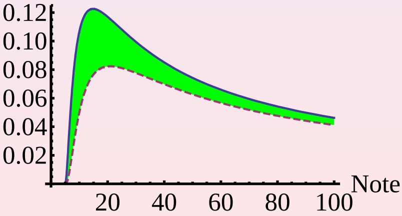
- In the example, $\kappa = q\chi'(v)/(v\Delta) = qBv^{B-1}/(v\Delta)$.
- Now, substitute equilibrium formulas (♣) for q and v:

$$\kappa \leq Bx^{1/A}\Delta^{-1+1/A}(1-\underline{\Delta}/\Delta)^{B+1/A-2}$$

- Passed rate $\pi = v\kappa$ lies below the counterfeiting rate, as seen in right graph below (empirical Euro passed rate at left, theoretical graph in example at right)

Counterfeiting and Passed Rates





The Hill-Shaped Counterfeit Rate

- Recalling the hot potato game, we have

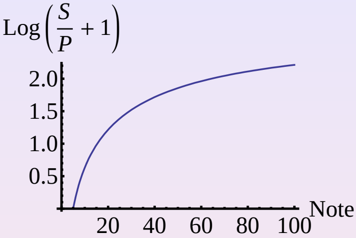
$$v\kappa\Delta = q\chi'(v) \quad \Rightarrow \quad \kappa = \frac{q\chi'(v)}{v\Delta}$$

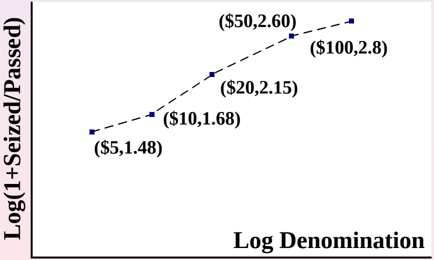
- **Theorem (The Hill-Shaped Counterfeit Rate).**
  (a) *The counterfeiting rate vanishes at lowest notes.*
  (b) *The counterfeiting rate vanishes also for the highest notes if marginal costs of quality $c'(q)$ explode.*
- Meanwhile quality explodes near highest notes, but not as fast as the denomination explodes, since its marginal cost rises: So $q/\Delta \to 0$ as $\Delta \to \infty$.

Seized Money and the Verification Rate

- Counterfeit money is eventually either *seized* from the criminals by law enforcement or the first verifiers, or successfully *passed* onto the public
- Seized money $S[\Delta]$ and passed money $P[\Delta]$
- steady-state condition 1:
  $$S[\Delta] + P[\Delta] = \text{value of counterfeit money found} = \text{counterfeit production}$$
- The inflow of passed money then equals the passing fraction times the counterfeit production:
  $$P[\Delta] = f(v[\Delta]) \cdot (\text{production value}) = f(v[\Delta]) \cdot (S[\Delta] + P[\Delta])$$
- So the *seized-passed ratio* $S[\Delta]/P[\Delta]$ obeys
  $$\frac{1}{f(v[\Delta])} = 1 + \frac{S[\Delta]}{P[\Delta]}$$
- Since $f(v)$ falls in v, and v rises in Δ, the seized-passed ratio should rise in Δ. Does it?

Rising Verification and Seized-Passed Ratio

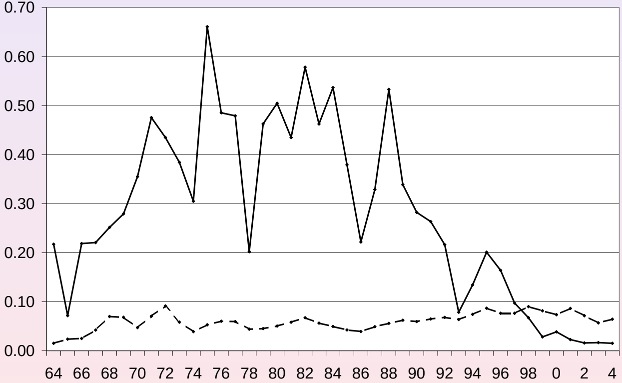




The Plummeting Counterfeit Seizures

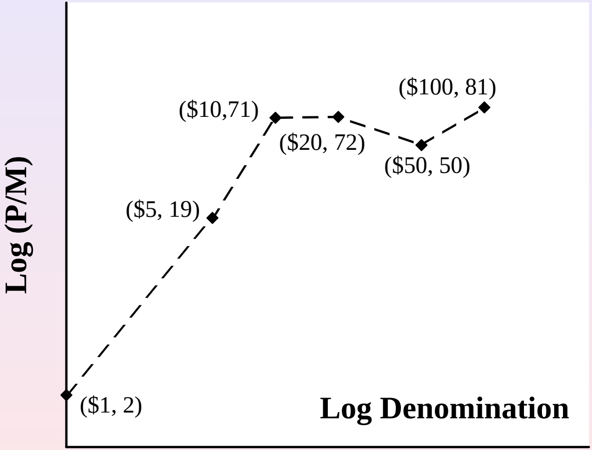
($\star$) Recall that technology reduces the verification rate.

Value of Domestic Passed & Seized Money Over Circulation.

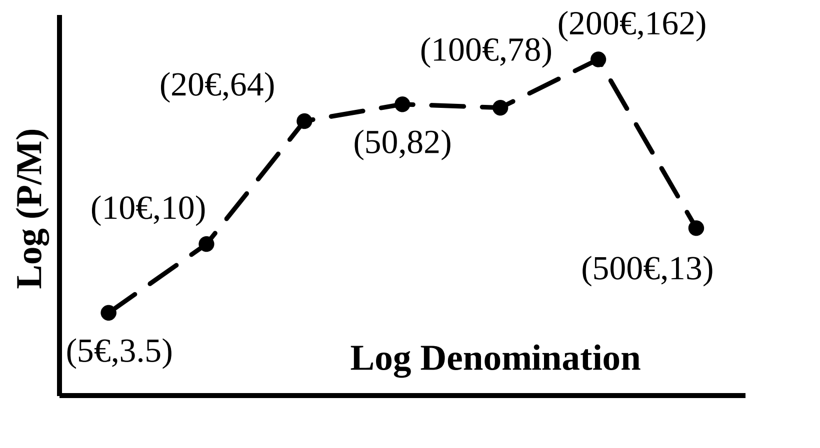


Logic: Technological revolution in counterfeiting.

Passed Counterfeit Money Rate, 1995–2007



Euro Passed Counterfeit Rate, 2002-5



Theoretical Links to the Literature

- The literature on counterfeiting is somewhat counterfactual, and caught up with the satisfaction of being general equilibrium, despite often finding no equilibria with counterfeiting.
- Costly verification or attention has been a large focus of much research (eg. Sims, 2003). This paper brings this thinking to the search and money literature. This is the first "behavioral" model in the field.
- Our model is a general equilibrium model when we view the verification rate as an implicit price. This is part of a line of research looking at "implicit markets."
- We think this is the first multi-market large game
- Our model is based on a large game, where payoffs depend on one's own action, the average action, and a state variable — eg. Angeletos and Pavan (2007). Unlike those models, our state variable is endogenous (the counterfeiting rate).