An Economic Theory Masterclass

Part V: Public Goods

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February 21, 2020

Public Goods Taxonomy

- *Rival* good: one consumer's use reduces another's benefit
- *Nonrival* good: no consumer's use reduces another's benefit
- *Excludable* / *nonexcludable* good: one can / cannot prevent others from jointly consuming a unit of the good

Goods	**Rival**	**Nonrival**
Excludable	Private good	Club good
Nonexcludable	Congestion public good	Pure public good

Examples of Public Goods

- Pure public goods
  - National defense, lighthouses, rural highways, AM/FM radio, drone airspace (Gatwick airport??)
  - Information goods (songs or movies, etc)
    - "He who receives an idea from me, receives instruction himself without lessening mine; as he who lights his taper at mine, receives light without darkening me." — Thomas Jefferson
- Congestion goods
  - city roads, all wifi (pre-2005), evening internet, due to video

HOME > DIGITAL > NEWS

OCTOBER 2, 2018 4:00AM PT

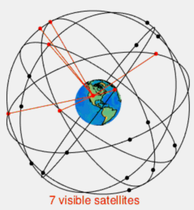
Netflix Eats Up 15% of All Internet Downstream Traffic Worldwide (Study)

Amazon Prime consumes more bandwidth than YouTube in Americas region: Sandvine report

- Club goods: satellite video, streaming video, golf courses, toll bridges and toll roads, satellite radio
- Wifi (post 2005)

GPS: Club Good to Pure Public Good

- '78: NAVSTAR Global Positioning System satellites launched.
- They circle the Earth at an altitude of 20,000 km and complete two orbits daily (not in a geostationary orbit)



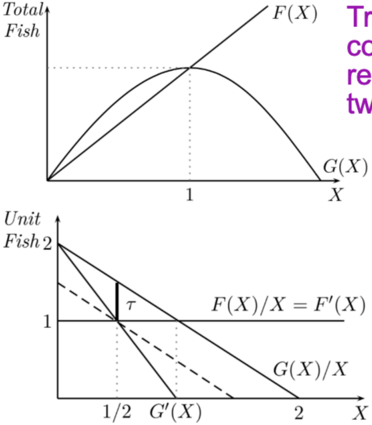
- 24 satellites ensure that ≥ 8 satellites can be simultaneously seen at any time from almost anywhere on Earth.
- May, 2000: govt stops degrading civilian GPS accuracy
- error $< 0.715m$, 95% of the time: The satellite atomic clocks
  - travel 14,000 km/hr $\Rightarrow$ tick 7 microseconds/day more slowly
  - face 4 times weaker gravity $\Rightarrow$ tick 45 microseconds /day faster
  - Without correcting for 38 microseconds per day due to relativity, navigational errors would exceed 10 km per day!!
- No Missiles! Smart phone GPS fails at high speed / altitude!

The Tragedy of the Commons

- Congestion public goods lead to the tragedy of the common
- Continuum mass M of fishermen each allocates hours X_A, X_B between Lakes A and B, where $X_A + X_B = \bar{X}$.
- Lake A has constant returns: $F(X_A) = X_A$
- Lake B has decreasing returns: $G(X_B) = 2X_B - X_B^2$
- Stable dynamics equalize returns on the lakes:
  - If $X_B > 1$, then $G(X_B)/X_B < 1 = F(X_A)/X_A$ return on Lake A, and fishermen exit Lake B.
  - If $X_B < 1$, then $G(X_B)/X_B > 1 = F(X_A)/X_A$ return on Lake A, and fishermen enter Lake B.
  - $F(X_A)/X_A = G(X_B)/X_B \Rightarrow 1 = 2 - \hat{X}_B \Rightarrow \hat{X}_B = 1$.
- Social planner: $\max F(X_A) + G(X_B)$ subject to $X_A + X_B = \bar{X}$
  - FOC equates the social marginal returns: $F'(X_A) = G'(X_B)$.

  $\Rightarrow$ $1 = 2 - 2X_B^* \Rightarrow X_B^* = 1/2 < 1 = \hat{X}$
  - The lake with diminishing returns is overfished
  - A Pigouvian tax $\tau = G(1/2)/(1/2) - G'(1/2) = 3/2 - 1 = 1/2$ decentralizes this efficient allocation

The Fishing Tragedy of the Commons

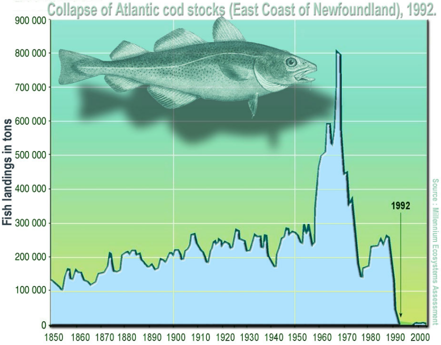


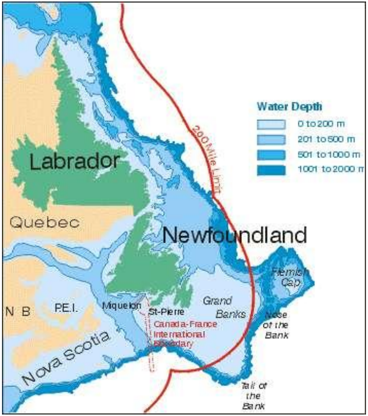
Tragedy of the commons: Average returns equalize on two lakes without taxes

Social planner equates marginal returns on lakes

- Individual decisions are inefficient because they are governed by the social average product and not social marginal product
- Example: drivers choose the congested highway and not the Waze back route if it is faster: They ignore the slightly increased driving time they inflict on thousands of others

The Fishing Tragedy of the Commons: Newfoundland





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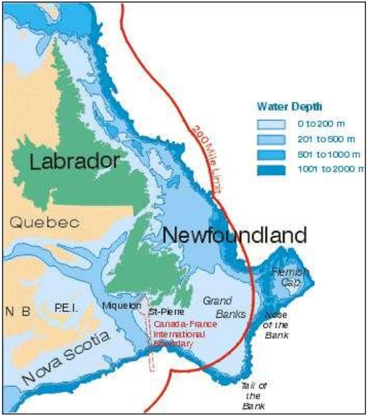
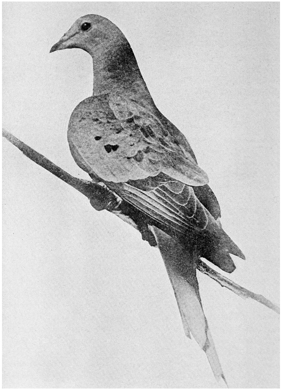


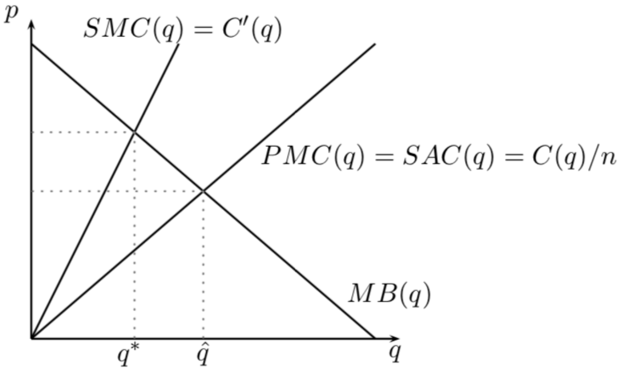
Cod off Newfoundland Before the Fishing Collapse

Migratory Birds and the Passenger Pigeon

- Martha, the last passenger pigeon, died on September 1, 1914, at the Cincinnati Zoo.
- The Migratory Bird Treaty Act, 1918 banned the possession of migratory birds for commercial purposes
- Even casting native bird species in movies is against the law!
- A "feather in your cap" is no longer allowed!

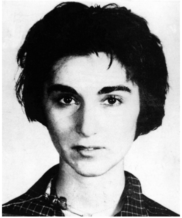


Group Dining Dilemma



- Assume an agreement or protocol to divide the check equally.
- Everyone then equates $MB = MC$, the *private marginal cost*.
- FOC is $MC = C/n < SMC$, the *social marginal cost*
- "Going Dutch", paying for their own meal $\Rightarrow MC = SMC$, and everyone chooses the efficient smaller meal $q^* < \hat{q}$.

Tragedy of Commons: The Bystander Effect



- Murder of Kitty Genovese on March 13, 1964
  - 38 witnesses saw or heard the attack, but none of them called the police or came to her aid

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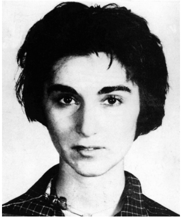

	no one else calls	someone else calls
call	a-c	-c
don't	b	a

a-c>b

a,b,c>0

- Murder of Kitty Genovese on March 13, 1964
  - 38 witnesses saw or heard the attack, but none of them called the police or came to her aid
  - Analyze this as a Nash equilibrium of *n* player game
- Modern day tragedy of commons: viral false news spreading

Efficient Provision Nonrival Discrete Public Goods

- Pure extensive margin exercise: Do we build a library?
- Individuals $i = 1, 2, \ldots, n$ have utility $U^i(G, m)$ increasing in amount G of public good and m of private good (money)
- Should we build it? Is $G = 1$ or $G = 0$
- Pareto Efficiency rule: $G = 1$ if $\exists$ *transfers* $t_1, \ldots, t_n$ from consumers paying for it ($\sum_i t_i \geq c$), such that
  - (a) everyone is weakly better off: $U^i(1, m_i - t_i) \geq U^i(0, m_i)$
  - (b) some j is strictly better off: $U^i(1, m_j - t_j) > U^i(0, m_j)$

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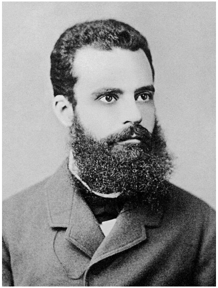
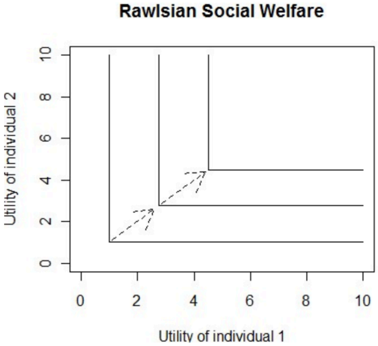
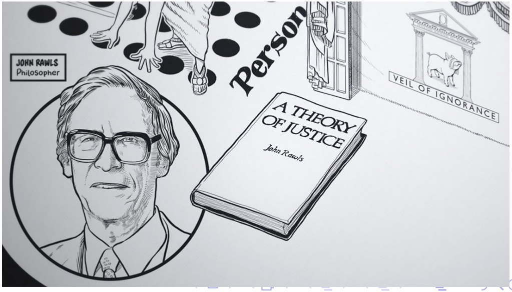
- Vilfredo Pareto, the fascist (1848–1923) $\Rightarrow$ social efficiency



Efficient Provision Nonrival Continuous Public Goods

- We now consider the question of how big to build the library
- With an intensive margin, we must trade off consumers' gains
- A social planner ("society") cares for all utilities $u^1, \ldots, u^n$.
- Society maximizes an increasing and quasi-concave (SWF) **social welfare function** $W(u^1, \ldots, u^n)$
- John Rawls (1921–2002) considered the extreme case of perfect complements SWF: $W(u^1, \ldots, u^n) = \min(u^1, \ldots, u^n)$.





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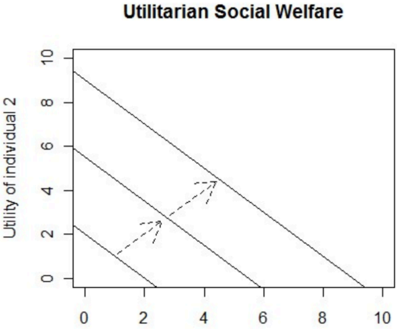
- Jeremy Bentham (1748–1832): "the greatest happiness of the greatest number is the foundation of morals and legislation"
- Perfect substitutes SWF: $W(u^1, \ldots, u^n) = u^1 + \cdots + u^n$



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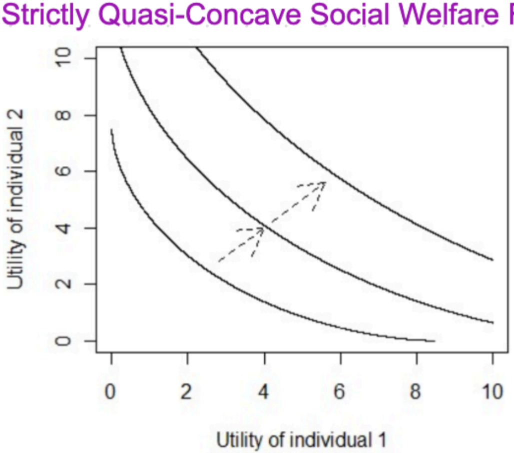
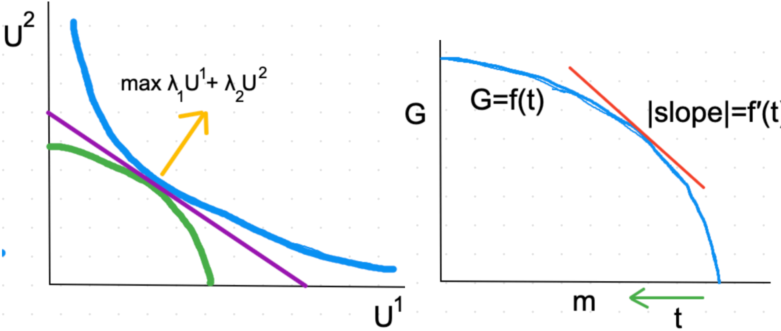


Efficient Provision Nonrival Continuous Public Goods

- Assume just two consumers, paying total transfer $t = t_1 + t_2$.
- Production function $G = f(t)$, where $f' > 0 > f''$.

$\Rightarrow$ **Claim:** $\exists$ strictly convex feasible utility set $\mathcal{U}$ for the planner

- $\max_{\{t_1,t_2\}} W(u^1, u^2)$ subject (u^1, u^2) feasible. By duality:



- Thus, it is sufficient to use a locally weighted utilitarian SWF:

$$\max_{\{t_1,t_2\}} \lambda_1 U^1(f(t_1 + t_2), m_1 - t_1) + \lambda_2 U^2(f(t_1 + t_2), m_2 - t_2)$$

Samuelson (1954), "The Theory of Public Expenditure"

$$\max_{\{t_1,t_2\}} \lambda_1 U^1(f(t_1+t_2), m_1-t_1) + \lambda_2 U^2(f(t_1+t_2), m_2-t_2)$$

$$\text{FOC} \quad \Rightarrow \quad \lambda_1 U^1_G f'(t) + \lambda_2 U^2_G f'(t) = \lambda_1 U^1_m = \lambda_2 U^2_m$$

- Divide first term by $\lambda_1 U^1_m f'(t)$ and second by $\lambda_2 U^2_m f'(t)$:

$$MRS^1_{G,m} + MRS^2_{G,m} = \frac{U^1_G}{U^1_m} + \frac{U^2_G}{U^2_m} = 1/f'(t) = MRT_{G,m}$$

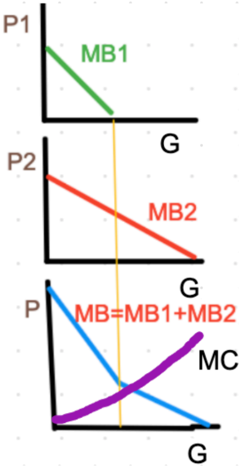
Lemma (The Samuelson Condition, 1954)

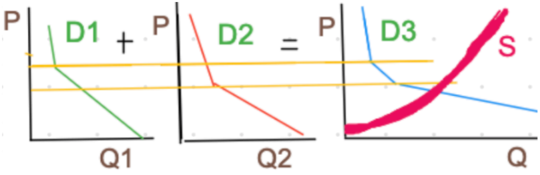
Optimal consumption of public good: $\sum_{i=1}^n MRS^i_{G,w} = MRT_{G,w}$.

- *Quasilinear preferences*: $U^i(G,w) = \phi_i(G) + w$
- Samuelson's Condition reduces to $\sum_{i=1}^n MB^i(G) = MC(G)$.

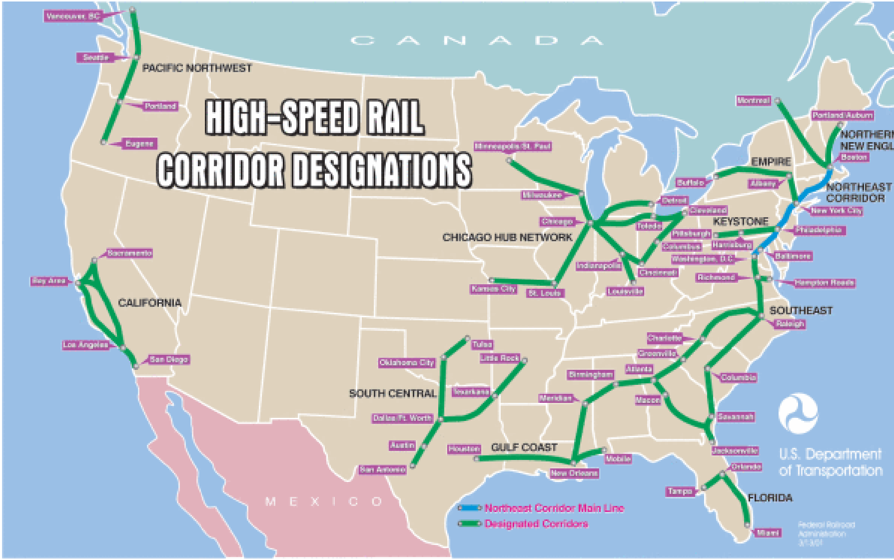
Efficiency with Private Goods vs Public Goods

- Public goods: common quantity and individual prices
- Private goods: common price and individual quantities





California Cancels $77B High Speed Rail Project



Lindahl Equilibrium

- The competitive equilibrium with public goods is inefficient.
- **In 1919**, Erik Lindahl decentralized the efficient outcome
- He devised a game (mechanism) whose unique Nash equilibrium is the efficient Samuelson public goods outcome

★ Nash (1950) and Samuelson (1954) came decades later!

- Eg.: How can n roommates efficiently pay for a Wi-fi router?
- A single private good x and a public good G
- Initial private good endowment $(w_1, \ldots, w_n)$
- Assume the public good is simply sold at a linear price p
- A **Lindahl Equilibrium** is a public and private goods allocation $(G^*, x_1^*, \ldots, x_n^*)$, and individual public good prices $(p_1, \ldots, p_n)$ with sum $p = p_1 + \cdots + p_n$, such that every consumer i chooses (G^*, x_i^*) given a price p_i for G:

$$(G^*, x_i^*) = \arg\max_{x_i, G} U^i(G, x_i) \quad \text{s.t.} \quad x_i + p_i G = w_i$$

- Knowing that he must pay a share p_i of the price p of the router, consumer i agrees on the public good G^*.

Theorem

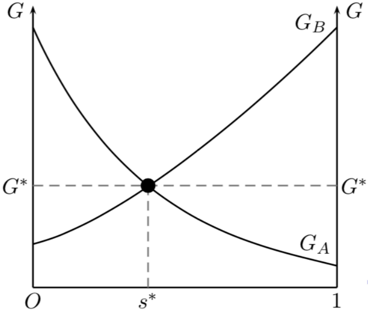
A Lindahl Equilibrium exists and is efficient.

- Intuition: Lindahl Equilibrium asks that individuals pay for the public good according to their marginal benefits
- *Proof*: FOC $\Rightarrow p_i = U^i_G / U^i_x = MRS^i_{Gx}$ for all $i = 1, \ldots, n$

$\Rightarrow \sum_i MRS^i_{Gx} = p_1 + \cdots + p_n = p = MRT_{Gx}$

$\Rightarrow$ Samuelson public goods efficiency condition holds.

- Proof depiction for two roommates $i = A, B$ choosing shares $s_i \geq 0$ to maximize $U^i(G, x_i)$ subject to $x_i + s_i pG = w$.
- Lindahl equilibrium requires $s_1 + s_2 = 1$



Lindahl Example

- Assume $U^A(G,x) = x^{1-\alpha} G^\alpha$ and $U^B(G,x) = x^{1-\beta} G^\beta$.
- Cobb Douglas: $G_A^* = \alpha w_A/(sp)$ and $G_B^* = \beta w_B/[(1-s)p]$.
- Finally, $G_A = G_B$ implies:

$$\frac{\alpha w_A}{sp} = \frac{\beta w_B}{(1-s)p} \quad \Rightarrow \quad s^* = \frac{\alpha w_A}{\alpha w_A + \beta w_B}$$

- A pays more for the more he likes Wifi and the wealthier he is.

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- A pays more for the more he likes Wifi and the wealthier he is.
- "With private goods, different people can consume different quantities, but in equilibrium they all must pay the same prices. With public goods, everyone must consume the same amount quantity, but in Lindahl equilibrium, they may pay different prices." — Ted Bergstrom

Peak Load Pricing

- We apply Lindahl equilibrium, where the different consumers are just the same consumer at different times.
- Assume peak and off-peak ferry service to Newfoundland
- Mid summer is peak ferry time, and off peak is spring and fall
- Inverse demand $p_H = h - X_H$ for peak season ferry tickets
- Off peak demand $p_L = \ell - X_L$, where $h > \ell$.
- Consumer surplus: $CS(X_L, X_H) = X_L^2/2 + X_H^2/2$
- Ferries annual loan cost, or *capacity cost*, is $\beta > 0$
- Ferry costs $b > 0$ to run (crew and fuel).
- Producer surplus for the capacity $\bar{X} \geq X_H, X_L$

$$PS(X_L, X_H) = (h - b - X_H)X_H + (\ell - b - X_L)X_L - \beta\bar{X}$$

Peak Load Pricing Solution

- Lagrangean:

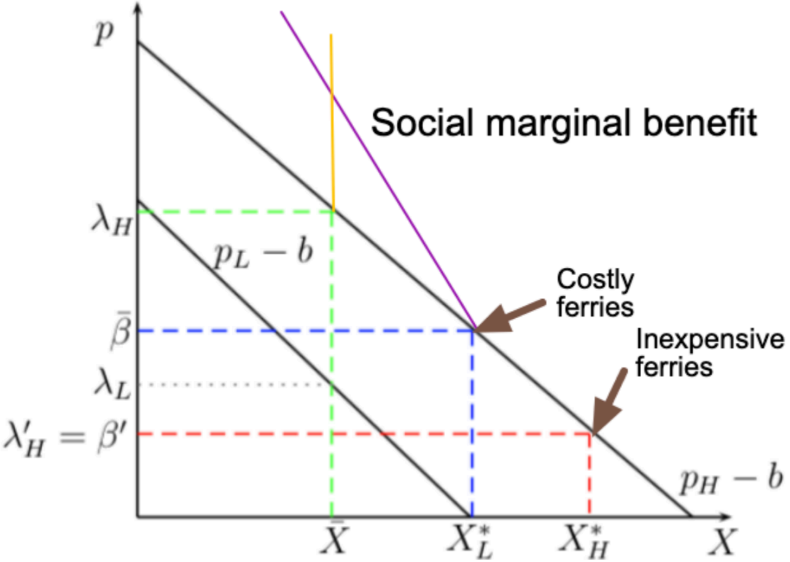
$$\mathcal{L} = CS(X_L, X_H) + PS(X_L, X_H) + \lambda_H(\bar{X} - X_H) + \lambda_L(\bar{X} - X_L)$$

- Kuhn Tucker conditions:

$$\begin{aligned} [X_H]: &\quad h - X_H - b = \lambda_H \\ [X_L]: &\quad \ell - X_L - b = \lambda_L \\ [\bar{X}]: &\quad \lambda_H + \lambda_L = \beta \\ [\lambda_H]: &\quad X_H \leq \bar{X}, \lambda_H \geq 0, \lambda_H(\bar{X} - X_H) = 0 \\ [\lambda_L]: &\quad X_L \leq \bar{X}, \lambda_L \geq 0, \lambda_L(\bar{X} - X_L) = 0 \end{aligned}$$

- Cheap ferries $\beta < \bar{\beta}$: Many are purchased, and not all are run at off-peak times; peak demand pays all ferry capital costs.
- Costly ferries $\beta > \bar{\beta}$:
  - Few are bought, and all run at off-peak times $X_L = X_H = \bar{X}$.
  - Both peak and off-peak pay for the ferries, namely, off-peak pays $\lambda_L > 0$ and peak pays $\lambda_H > 0$, where $\lambda_L + \lambda_H = \beta$.

Peak Load Pricing Solution



The Ferry to Newfoundland

The Net Usually Says No to Peak Load Pricing