An Economic Theory Masterclass

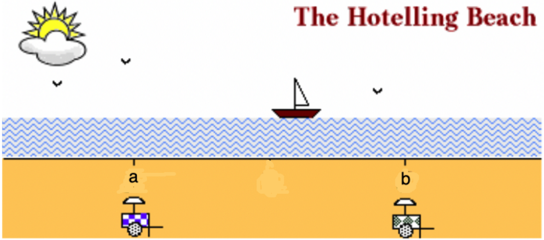
Part IX: General Equilibrium with Spatial Competition

Lones Smith

March 5, 2020

The Hotelling Model

- Harold Hotelling (1929), "Stability in Competition", *EJ*



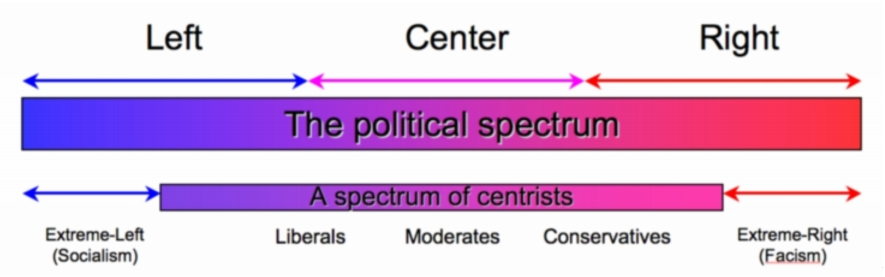
- Ike and Joe each own lemonade pushcart along a unit beach.
- Ike is located at a and Joe at b, where $0 \leq a \leq b \leq 1$.
- Lemonade is \$2 per glass, *by fiat.*
- Customers are located evenly along beach $[0, 1]$
  - have willingness to pay $v > 1$ for a single cup of lemonade
  - Buyer $x \in [0, 1]$ pays transportation cost $|x - a|$ to walk to a
  - Total sales are independent of where sellers locate (as $v > 1$)

Principle of Minimum Differentiation

- Given an equal sharing tie break rule if Ike and Joe locate at the same spot, the unique Nash equilibrium is $a = b = 1/2$.
- When Hotelling added a price setting subgame, firms wish to move away from each other. [d'Aspremont, Gabszewicz and Thisse (1979) famously corrected Hotelling, fifty years later!]

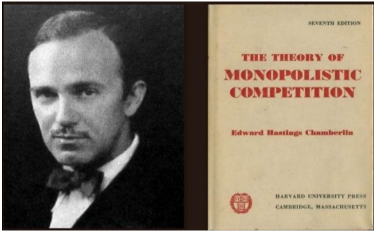
Principle of Minimum Differentiation

- Given an equal sharing tie break rule if Ike and Joe locate at the same spot, the unique Nash equilibrium is $a = b = 1/2$.
- When Hotelling added a price setting subgame, firms wish to move away from each other. [d'Aspremont, Gabszewicz and Thisse (1979) famously corrected Hotelling, fifty years later!]
- Lacking prices, it is used more as a location metaphor in a left-right political spectrum, and explained why the movements toward the center are predicted.
  - If entry is allowed, then this explains the appearance of extreme left and right third parties



Chamberlin's Monopolistic Competition

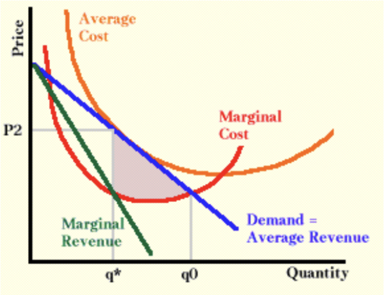
- Chamberlin, *A Theory of Monopolistic Competition* (1933)



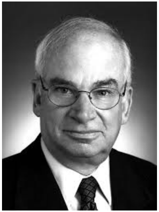
- Chamberlin coined the term "product differentiation"
- both price and location competition.
- If two sellers were very close, say near $x = 1/2$, then each seller raises its demand by moving away from the other.
- Why? That lowers the transportation costs for a larger mass of consumers than it raises transportation costs for.

Monopolistic Competition

- Transportation costs ⇒ each firm has a falling demand curve
- With free entry, firms enter if they can cover their fixed costs.
- Price then exceeds marginal cost when profits vanish at just one quantity q^* (demand curve is tangent to average cost)
- This is really just a model of Bertrand-Nash price competition: since firms have falling demand curves, it is not competitive
- Example: In the economics textbooks market, a small slice of the principles textbook market, you are set for life as a millionaire: Mankiw (!!), Bernanke, Krugman.

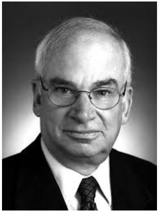


Rosen's Competitive Model of Hedonic Pricing



- Rosen (1974): With small fixed costs, competitive price taking behavior is a better model of product differentiation
- Goods vary by attribute — size, power, weight, location

Rosen's Competitive Model of Hedonic Pricing



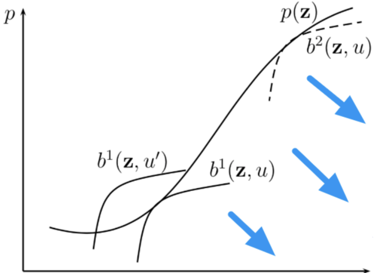
- Rosen (1974): With small fixed costs, competitive price taking behavior is a better model of product differentiation
- Goods vary by attribute — size, power, weight, location
  - for houses, what matters most is "location, location, location"
- How does a car price vary with size, power, weight, or an apartment price vary with location?
- Hedonic prices are the implicit prices of attributes, as revealed by the observed prices of differentiated products.
- A market-clearing competitive price function $p(\mathbf{z}) = p(z_1, \ldots, z_n)$ reflects characteristics $\mathbf{z}$

The Consumer's Spatial Problem

- Utility $U(x, \mathbf{z})$ depends on money x and $\mathbf{z} = (z_1, \ldots, z_n)$.
- The consumer with utility U and money income y solves

$$\max_{(x,\mathbf{z})} U(x, \mathbf{z}) \text{ s.t. } x + p(\mathbf{z}) = y$$

- Thus, *he takes the price function as given* — i.e. competition
- The bid function $b(\mathbf{z}, \bar{u})$ solves $U(y - b, z_1, \ldots, z_n) \equiv \bar{u}$.
- Indifference curve $U(y - b, \mathbf{z}) \equiv \bar{u}$ has MRS $b_{z_i}(\mathbf{z}, \bar{u}) = U_{z_i}/U_x$.
- FOC: Bid function is tangent to the price function $b_{z_i} = p_{z_i}$
- Price function is the upper envelope of the bid functions.

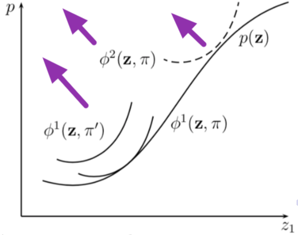


The Firm's Spatial Problem

- Rosen studies *short run equilibrium where firm's type is fixed*
- $C(Q, \mathbf{z})$ = cost of quantity Q of good $\mathbf{z} = (z_1, \ldots, z_n)$.
- In the *long run*, the firm chooses Q and $\mathbf{z}$ to maximize profits

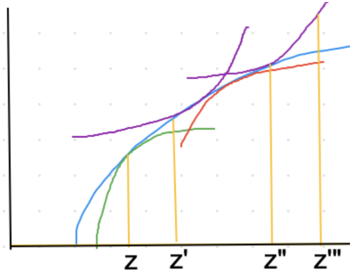
$$\max_{Q,\mathbf{z}} \Pi(p, Q, \mathbf{z}) = Qp(\mathbf{z}) - C(Q, \mathbf{z})$$

- In other words, *it takes the price function as given.*
- FOC in Q: $p(\mathbf{z}) = C_Q(Q, \mathbf{z}) \Rightarrow$ supply function $Q^* = Q^*(p, \mathbf{z})$
- FOC in z: $\Pi_{z_i}(p, Q^*, \mathbf{z}) = 0$ for all i yields $p_{z_i} = C_{z_i}/Q^*$.
- Offer function $\phi(\mathbf{z}, \bar{\pi})$ solves $\Pi(\phi(\mathbf{z}, \bar{\pi}), Q^*(p, \mathbf{z}), \mathbf{z}) \equiv \bar{\pi}$.
- FOC: Offer function is tangent to the price function $b_{z_i} = p_{z_i}$
- Price function is the lower envelope of the offer functions.



Market Equilibrium

- Market equilibrium is a price function $p(\mathbf{z})$, demand density $D(\mathbf{z})$, and supply density $S(\mathbf{z})$, with $D(\mathbf{z}) \equiv S(\mathbf{z})$ for all $\mathbf{z}$.
- For quality changes, the slope of the market price function reflects the value of quality change of no particular consumer.
  - $p(z') - p(z)$ *overstates* the value of the quality change for a consumer who buys z, and understates the value of the quality change for consumers who buy z'.
  - $p(z''') - p(z'')$ *understates* the cost of quality improvement for producers who sell z'', and overstates the cost of quality improvement for producers who sell z'''.



Two Location Hedonic Example

- Live next to the Capitol ($z = 1$), or far from it ($z = 0$)
- The competitive rent at $z = 0$ is fixed at $r > 0$, but there is an endogenous premium rent $R > r$ at $z = 1$
- Mass μ of residents has distaste $\theta \in [0, \mu]$ for Capitol
- Ms. θ has utility $U(x, z|\theta) = x + z/\theta$ over locale z & money x
- Height h costs $C(h) = L + h^2$, given land cost premium $L > 0$.

Hedonic Example Solution

- Mass $\bar{\theta}$ of residents $\theta \in [0, \bar{\theta}]$ live at $z = 1$, for some $\bar{\theta} > 0$
- A spatial competitive equilibrium $(\bar{\theta}, h, m, R)$:
  (1) Buildings at location 0 earn zero profits: $L + h^2 = C(h) = hR$
  (2) Each building's height is optimal: $2h = C'(h) = R$
  (3) *Resident type $\bar{\theta}$ is indifferent:* $R = r + 1/\bar{\theta}$
  (4) Apt. market clears at $z = 1$: $h = \bar{\theta} =$ resident mass in $[0, \bar{\theta}]$
- Solving the four equations in four unknowns:
  - From (1) and (2): $L = h^2 \Rightarrow h = \sqrt{L}$, $R = 2\sqrt{L}$
  - From (3): $1/\bar{\theta} = R - r = 2\sqrt{L} - r$
  - From (4): $\bar{\theta} = h = \sqrt{L}$
  - Solution: $\sqrt{L} = r + \sqrt{r^2 + 8}$
    $\bar{\theta} = h = r + \sqrt{r^2 + 8}$
  - $R = 2r + 2\sqrt{r^2 + 8}$
- So the Capitol land cost premium L rises as the square of the regular land rental r, leading to taller apartments built, charging a higher rent premium R
- Hence, Manhattan has very tall buildings and insane rents

PhD

Transportation Problem $\longrightarrow$ Transporter

My Girl