An Economic Theory Masterclass

Part IX: General Equilibrium with Spatial Competition

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The Hotelling Model

Harold Hotelling (1929), “Stability in Competition”, *EJ*

Ike and Joe each own lemonade pushcart along a unit beach.

- Ike is located at $a$ and Joe at $b$, where $0 \leq a \leq b \leq 1$.
- Lemonade is $2$ per glass, by fiat.
- Customers are located evenly along beach $[0, 1]$.
  - have willingness to pay $v > 1$ for a single cup of lemonade
  - Buyer $x \in [0, 1]$ pays transportation cost $|x - a|$ to walk to $a$
  - Total sales are independent of where sellers locate (as $v > 1$)
Principle of Minimum Differentiation

- Given an equal sharing tie break rule if Ike and Joe locate at the same spot, the unique Nash equilibrium is $a = b = 1/2$.
- When Hotelling added a price setting subgame, firms wish to move away from each other. [d’Aspremont, Gabszewicz and Thisse (1979) famously corrected Hotelling, fifty years later!]
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▶ Lacking prices, it is used more as a location metaphor in a left-right political spectrum, and explained why the movements toward the center are predicted.
  ▶ If entry is allowed, then this explains the appearance of extreme left and right third parties
Chamberlin’s Monopolistic Competition

- Chamberlin, *A Theory of Monopolistic Competition* (1933)

- Chamberlin coined the term “product differentiation”
- both price and location competition.
- If two sellers were very close, say near $x = 1/2$, then each seller raises its demand by moving away from the other.
- Why? That lowers the transportation costs for a larger mass of consumers than it raises transportation costs for.
Monopolistic Competition

- Transportation costs ⇒ each firm has a falling demand curve
- With free entry, firms enter if they can cover their fixed costs.
- Price then exceeds marginal cost when profits vanish at just one quantity $q^*$ (demand curve is tangent to average cost)
- This is really just a model of Bertrand-Nash price competition: since firms have falling demand curves, it is not competitive
- Example: In the economics textbooks market, a small slice of the principles textbook market, you are set for life as a millionaire: Mankiw (!!), Bernanke, Krugman.
Rosen’s Competitive Model of Hedonic Pricing

- Rosen (1974): With small fixed costs, competitive price taking behavior is a better model of product differentiation
- Goods vary by attribute — size, power, weight, location
Rosen’s Competitive Model of Hedonic Pricing

Rosen (1974): With small fixed costs, competitive price taking behavior is a better model of product differentiation

Goods vary by attribute — size, power, weight, location
  ▶ for houses, what matters most is “location, location, location”

How does a car price vary with size, power, weight, or an apartment price vary with location?

Hedonic prices are the implicit prices of attributes, as revealed by the observed prices of differentiated products.

A market-clearing competitive price function $p(z) = p(z_1, \ldots, z_n)$ reflects characteristics $z$
The Consumer’s Spatial Problem

- Utility $U(x, z)$ depends on money $x$ and $z = (z_1, \ldots, z_n)$.
- The consumer with utility $U$ and money income $y$ solves

$$\max_{(x, z)} U(x, z) \text{ s.t. } x + p(z) = y$$

- Thus, he takes the price function as given — i.e. competition
- The bid function $b(z, \bar{u})$ solves $U(y - b, z_1, \ldots, z_n) \equiv \bar{u}$.
- Indifference curve $U(y - b, z) \equiv \bar{u}$ has MRS

$$b_{zi}(z, \bar{u}) = \frac{U_{zi}}{U_x}.$$ 

- FOC: Bid function is tangent to the price function $b_{zi} = p_{zi}$
- Price function is the upper envelope of the bid functions.
The Firm’s Spatial Problem

- Rosen studies short run equilibrium where firm’s type is fixed
- $C(Q, z) =$ cost of quantity $Q$ of good $z = (z_1, \ldots, z_n)$.
- In the long run, the firm chooses $Q$ and $z$ to maximize profits

$$\max_{Q, z} \Pi(p, Q, z) = Qp(z) - C(Q, z)$$

- In other words, it takes the price function as given.
- FOC in $Q$: $p(z) = C_Q(Q, z) \Rightarrow$ supply function $Q^* = Q^*(p, z)$
- FOC in $z$: $\Pi_{z_i}(p, Q^*, z) = 0$ for all $i$ yields $p_{z_i} = C_{z_i}/Q^*$.
- Offer function $\phi(z, \bar{\pi})$ solves $\Pi(\phi(z, \bar{\pi}), Q^*(p, z), z) \equiv \bar{\pi}$.
- FOC: Offer function is tangent to the price function $b_{z_i} = p_{z_i}$
- Price function is the lower envelope of the offer functions.
Market Equilibrium

- Market equilibrium is a price function \( p(z) \), demand density \( D(z) \), and supply density \( S(z) \), with \( D(z) \equiv S(z) \) for all \( z \).

- For quality changes, the slope of the market price function reflects the value of quality change of no particular consumer.
  - \( p(z') - p(z) \) overstates the value of the quality change for a consumer who buys \( z \), and underestimates the value of the quality change for consumers who buy \( z' \).
  - \( p(z''') - p(z'') \) understates the cost of quality improvement for producers who sell \( z'' \), and overstates the cost of quality improvement for producers who sell \( z''' \).
Two Location Hedonic Example

- Live next to the Capitol \((z = 1)\), or far from it \((z = 0)\)
- The competitive rent at \(z = 0\) is fixed at \(r > 0\), but there is an endogenous premium rent \(R > r\) at \(z = 1\)
- Mass \(\mu\) of residents has distaste \(\theta \in [0, \mu]\) for Capitol
- Ms. \(\theta\) has utility \(U(x, z|\theta) = x + z/\theta\) over locale \(z\) & money \(x\)
- Height \(h\) costs \(C(h) = L + h^2\), given land cost premium \(L > 0\).
Hedonic Example Solution

- Mass $\bar{\theta}$ of residents $\theta \in [0, \bar{\theta}]$ live at $z = 1$, for some $\bar{\theta} > 0$
- A spatial competitive equilibrium $(\bar{\theta}, h, m, R)$:
  1. Buildings at location 0 earn zero profits: $L + h^2 = C(h) = hR$
  2. Each building’s height is optimal: $2h = C'(h) = R$
  3. Resident type $\bar{\theta}$ is indifferent: $R = r + 1/\bar{\theta}$
  4. Apt. market clears at $z = 1$: $h = \bar{\theta} = \text{resident mass in } [0, \bar{\theta}]$

- Solving the four equations in four unknowns:
  - From (1) and (2): $L = h^2 \Rightarrow h = \sqrt{L}, \ R = 2\sqrt{L}$
  - From (3): $1/\bar{\theta} = R - r = 2\sqrt{L} - r$
  - From (4): $\bar{\theta} = h = \sqrt{L}$
  - Solution: $\sqrt{L} = r + \sqrt{r^2 + 8}$
    $\bar{\theta} = h = r + \sqrt{r^2 + 8}$
    $R = 2r + 2\sqrt{r^2 + 8}$

- So the Capitol land cost premium $L$ rises as the square of the regular land rental $r$, leading to taller apartments built, charging a higher rent premium $R$
- Hence, Manhattan has very tall buildings and insane rents
Transportation Problem \( \rightarrow \) Transporter