The Behavioral SIR Model, with Applications to the Swine Flu and COVID-19 Pandemics

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Virtual Macro Presentation on 4/10 by Lones Smith (Views expressed are not those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.)



- Contagion math in the best of times depends on
  - 1. Biology: how infectious is the infection?
  - 2. Sociology: how networked we are; how segregated it is "Super spreaders" (Giuseppe Moscarini)
  - 3. Geography: meeting rates are higher in dense cities
  - 4. Culture: in Italy, the kiss sometimes replaces the handshake
  - 5. Game theory: how we react to payoffs and each other
  - 6. Political economy: how fast/major is political action (like shutdowns)? Are people responsive?

For Whom the Bell Tolls: Avoidance Behavior at Breakout in COVID19

SI / SIS / SIR



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# The SIR Model (1927)

- ▶ The model takes place in periods 1, 2, . . ..
- ▶ Population is viewed as the continuum [0,1]
- State transition process of people in the SIR model:
   0. mass σ is susceptible, then if one gets infected
  - 1. mass  $\pi$  is *infected* / contagious but asymptomatic/oblivious,
  - $2. \ \mbox{infected} \ / \ \mbox{contagious symptomatically so, and not meeting,}$
  - 3. mass  $\rho$  is *recovered/removed* and immune (sickness /death)

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# The SIR Model (1927)

- The contagious share  $\pi \in (0,1)$  is called the *prevalence*
- A contagious person infects a random number susceptible people each period with (mean) β > 0, called the *passage rate*
- ► *Incidence*, or inflow of new infections, is  $\beta \sigma \pi$  assuming random and independent meetings
- Anyone infected "recovers" (or dies / is removed from the infected pool) at recovery rate r > 0.

# The SIR Model

The SIR Model implies the general time dynamics (daily changes) in susceptible and infected shares:

$$\dot{\sigma}(t) = -incidence = -eta \pi(t) \sigma(t)$$

$$\dot{\pi}(t) \;\;=\;\;$$
 incidence  $-$  recoveries  $=eta\pi(t)\sigma(t)-r\pi(t)$ 

$$\dot{
ho}(t)$$
 = recoveries =  $r\pi(t)$ 

- So susceptible share  $\sigma(t)$  always falls.
- Infected share  $\pi(t)$  first rises and then falls.
- We can safely ignore  $\pi(t)$  since it does not impact dynamics.

## Endgame

- Epidemic dies out
  - $\Leftrightarrow$  the susceptible share  $\sigma$  low enough
  - $\Leftrightarrow \mathsf{recovered}/\mathsf{immune} \ \mathsf{fraction} \ \rho \ \mathsf{high} \ \mathsf{enough}$
- Herd immunity tipping point:  $\Leftrightarrow$  incidence equals recoveries  $\Leftrightarrow$  infection inflow balances outflow  $\Leftrightarrow \beta \hat{\sigma} \hat{\pi} = r \hat{\pi}.$
- ▶ Since susceptibles falls,  $\sigma \leq \bar{\sigma}$  thereafter: contagion vanishes

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- $\blacktriangleright$  Since susceptibles falls,  $\sigma \leq \bar{\sigma}$  thereafter: contagion vanishes
- $\Rightarrow$  Define **R0**  $\equiv \beta/r$ .
- Herd immunity  $101 \Leftrightarrow \beta \sigma \pi \leq r \pi \Leftrightarrow \sigma \cdot R0 \leq 1$ .
- ▶ Published COVID estimates  $R0 = 2.3 \Rightarrow \rho_t > 1 1/2.3 \approx 0.56$
- "Newsom projection: 56% of California would be infected in 8 weeks without mitigation effort" (2020/03/19)

## Incentives Matter in Contagions

- ▶ We will fix biology (focus on H1N1 and later COVID19)
- We ignore geography and culture since they do not change in the course of the contagion
- We ignore political economy for Swine Flu (no serious public actions emerged), but not COVID19

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- We ignore geography and culture since they do not change in the course of the contagion
- We ignore political economy for Swine Flu (no serious public actions emerged), but not COVID19
- We dispute the absolute meaning of passing rates or R0. These respond to incentives.
  - Example: Measles outbreaks have much higher R0 than measles pandemics.
- We will focus on optimizing strategic behavior, since
  - it can change very rapidly in the contagion
  - and we show that it does

## Incentives Matter in Contagions

- A disease passes the same among humans or animals in the SIR model.
- But homo economicus adjusts behavior to avoid sickness or death:
- Historically, behavior has changed, like quarantines off Venice in the 1300s during Black Death
- 1980s HIV/AIDS increased "safe sex" efforts: condoms or check your partners history
- In meetings that do occur, people wash their hands, or sneeze into elbows, or weak masks

## Research Background

- Geoffard & Philipson (1996), "Rational Epidemics and Their Public Control" (*IER*, 1996), introduced rational avoidance optimization into an AIDS matching model
- "Economics of Counterfeiting" (2004/2010 with Quercioli)
- $\hookrightarrow$  "Contagious Matching Games" (2006 w/ Quercioli), WP only presented at Penn S&M Conference
- Greenwood, Kircher, Cezar & Tertilt. "An Equilibrium Model of the African HIV/AIDS Epidemic" (*Econometrica*, 2019)

The Contagion Game

▶ World's biggest game: Everybody\* in the world is a player.

The highest stake game: life of death (or sickness): loss L

• Vigilance  $v \ge 0$  is the action in the game.

# Filter function

- Filter function  $f(v) \in [0, 1]$  linearly scales down passage rates
- ⇒ Passage rate is  $\beta f(v)f(w)$  if vigilance v contagious person just meets vigilance w susceptibles
  - $\Rightarrow$  diminishing returns:  $f(0)=1>0=f(\infty)$  & f'<0< f''.
  - A symmetric function is a simplifying assumption
    - a mask is equally protective of both parties.
    - Not meeting also symmetrically protects both parties f(v) = fraction of meetings one keeps
  - This multiplicative (log-modular) form is for simplicity.
  - A vaccination is easy vigilance: one jab  $\Rightarrow$  nearly perfect filter

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  - A vaccination is easy vigilance: one jab  $\Rightarrow$  nearly perfect filter
  - ▶ Posit hyperbolic filter function  $f(v) = (1 + \zeta v)^{-\gamma}$ , for  $\gamma > 0$ 
    - $1/\zeta \approx contagiousness$  (more vigilance effort needed as  $\zeta$  falls)
    - $\gamma$  filter responsiveness  $\approx$  proportionate fall in passage rate from vigilance
      - $\hookrightarrow$  should vary by population density, higher in cities

# Vigilance Optimization

For any common vigilance  $\bar{v}$ , his period *infection chance* is

infection rate = (passage rate) × (prevalence)

An unsure person (susceptible or asymptomatic infected) is susceptible with chance q(π) = σ/(σ + π). So updating:

 $\iota(\mathbf{v}|\bar{\mathbf{v}}) = \beta f(\mathbf{v}) f(\bar{\mathbf{v}}) q(\pi) \pi$ 

Infection chance falls when v or v
 ises, or prevalence π falls.
 Everyone minimizes

Expected Total Losses  $= \iota(\mathbf{v}|\bar{\mathbf{v}})\mathbf{L} + \mathbf{v}$ 

- Marginal analysis works:  $f' < 0 < f'' \Rightarrow \iota'(v|\bar{v}) < 0 < \iota''(v|\bar{v})$ .
- We assume complete information! To properly optimize, everyone must know the prevalence π and losses L.
- ▶ PS: It would be terrible if a public authorities lowballed  $\pi$  or L

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# Individual Optimality

If the optimal vigilance is v<sup>\*</sup> > 0, then marginal benefit equals marginal cost: −ℓ'(v<sup>\*</sup>|v̄)L = 1

 $\textbf{MB} = \textbf{Marginal Decrease in Infection Chance} \times \textbf{Loss} = \textbf{MC}$ 

• Marginal reduction in infection chance  $-\iota'(v^*|\bar{v})$  falls when v or  $\bar{v}$  rises, or prevalence  $\pi$  falls.

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#### Nash Equilibrium of a Game of Strategic Substitutes



Nash equilibrium. Everyone optimizes in response to others.

► Vigilance choices are substitutes ⇒ optimal vigilance v\* falls when v rises.

## Privately Optimal Vigilance Rises if Prevalence Rises



Privately optimal vigilance rises in prevalence.

#### Strategic Relaxation with Rising Prevalence



Equilibrium vigilance rises in prevalence less than privately optimal vigilance.

# The Behavioral SIR model (BSIR)

- Because f'(0) < 0 is finite, the marginal benefit at vigilance v = 0 is less than unit marginal cost for low prevalence π > 0.
  - Intuitively, I don't change my behavior at all unless the risk exceeds the background risk
- ► Nash equilibrium of the Contagion Game, rising vigilance  $v^*$  slowly chokes off the passage rate after  $\pi \ge \underline{\pi} \approx (\beta \zeta \gamma L)^{-1}$
- Define inverse to the filter responsiveness  $\varphi = 1/(1+2\gamma)$
- Nash equilibrium

BSIR infection chance 
$$\approx \begin{cases} \beta \pi & \pi \leq \underline{\pi} & (Chill) \\ \underline{\pi}^{1-\varphi}\beta \pi^{\varphi} & \pi > \underline{\pi} & (Vigilant) \end{cases}$$

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$$\text{BSIR passage rate} \ \approx \ \begin{cases} \beta & \pi \leq \underline{\pi} \quad \text{(Chill)} \\ (\underline{\pi}/\pi)^{1-\varphi}\beta & \pi > \underline{\pi} \quad \text{(Vigilant)} \end{cases}$$

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## Behavioral SIR Infection Chance (BSIR)



Figure: **Equilibrium Infection Chance Under the BSIR**. SIR Dynamics hold for a smaller prevalence interval  $[\underline{\pi}, 1]$  if the disease is more dire. E.g. For the common flu, SIR dynamics may obtain over a large prevalence interval.

#### Behavioral SIR Incidence Prevalence Elasticity

#### Lemma

Except near zero prevalence, the infection chance - prevalence elasticity is  $\varphi = 1/(1+2\gamma)$ . Thus, near breakout when  $\sigma \approx 1$ :

 $log(incidence) = constant + \varphi \cdot log(prevalence)$ 

- $\Rightarrow$  1% higher prevalence increases incidence by only arphi%
- We will simply call  $\varphi$  the *prevalence elasticity*

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#### Corollary (Size Matters)

Vigilance and thus the passing rate are both endogenous to nation size: For the same number of cases, other things equal, since the prevalence  $\pi$  is lower, the passage rate is higher.

Comment: We focus on SIR dynamics, but the strategic optimization logic transfers unchanged to SI and SIS models: the incidence prevalence elasticity is less than one

## Is the Toy Model Robust?

- We assumed a static representative agent model.
- ► First, the Behavioral SIR emerges with heterogeneity
  - We assume random and independent meetings.
  - But if there are high loss and low loss individuals, stopping them from meeting (eg. "seniors only shopping hour") yields two simultaneous disease stories going on.
- We have put aside forward-looking behavior

#### **Expected Total Losses** = $\iota(\nu | \bar{\nu})$ (DISEASE LOSS) + $\nu$

- Without discounting the future, averting a loss now may just delay the "inevitable", there is no gain.
- Fatalism: The more you care about the future, the more you care about your eventual demise, and the greater your disease loss — your efforts just delay the "inevitable"
- With discounting, incentives weaken in
- With one or two year pandemics that only kill some people, vigilance ramps up near the end, as the survival chance rises

## Behavioral SIR Dynamics Valid for all Times

- Assume an initial infection seed  $\pi_0 > 0$ 
  - Typhoid Mary moving from New York to NYC
  - Possibly farmers arriving with Swine Flu
  - People off airplanes infected with COVID-19
- Since  $\sigma \approx 1$  near *breakout*, the SIR dynamics imply

 $\dot{\pi}(t) = \text{incidence-recoveries} \approx \beta \pi(t) - r \pi(t) \quad \Rightarrow \quad \underline{\pi} \approx \pi_0 e^{(\beta - r)t}$ 

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- ▶ It makes sense to assume heightened vigilance for all  $t \ge \underline{\tau}$ , even after the contagion slows down.
- $\Rightarrow$  For times  $t \geq \underline{\tau}$ , Behavioral SIR dynamics are

$$\begin{aligned} \dot{\sigma}(t) &= -B(t)\pi(t)^{\varphi}\sigma(t) \\ \dot{\pi}(t) &= B(t)\pi(t)^{\varphi}\sigma(t) - r\pi(t) \\ \dot{\rho}(t) &= r\pi(t) \end{aligned}$$

for behavioral passage rate  $B(t) = \left(\frac{\pi(t) + \sigma(t)}{\sigma(t)}\right)^{1-\varphi} \underline{\pi}^{1-\varphi}\beta < \beta$ 

## BSIR Herd Immunity: Susceptible/Recovered Time Paths







## Endgame: Behavioral Herd Immunity

Now, consider the dire herd immunity endgame when inflow balances outflow: βč<sub>φ</sub> π̃<sup>φ</sup><sub>φ</sub> = rπ̃<sub>φ</sub>, or

$$\check{\sigma}_{arphi} = (\mathbf{r}/eta)/\check{\pi}_{arphi}^{1-arphi} > \mathbf{r}/eta = \hat{\sigma}$$

 $\Rightarrow$  Herd immunity happens at a higher susceptible share of the population  $\Leftrightarrow$  lower immune share

#### Theorem (Herd Immunity)

As the prevalence elasticity  $\varphi$  falls below 1, the herd immunity threshold  $1 - \sigma_{\varphi}$  falls.

#### Falling Herd Immunity in Passing Elasticity



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# Empirical Analysis of Swine Flu Pandemic in USA

▶ We acquired data from 41 states, for weekly or daily data

• One major state refused to release data.

We use the daily data of NY, TX, ME, GA, NJ, CO, MI, MA

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## The Behavioral SIR Model: Evidence from H1N1 (2009)



## Herd Immunity End Game for Swine Flu

CDC estimates 60.8 million cases of H1N1, or 19.8%

- ▶ Spring 2009: CDC forecast  $\approx 2/3$  of Americans would get infected.
- $\Rightarrow$  Herd immunity threshold was about 15% immune by infection and 5% immune by vaccination
- ► H1N1 vanished when flu normally rages in December.
- Herd immunity threshold was very low!

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#### Swine Flu ended by Minimal Vaccination



For Whom the Bell Tolls: Avoidance Behavior at Breakout in COVID19

#### Swine Flu ended by Minimal Vaccination



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## For Whom the Bell Tolls ...

"No man is an island, entire of itself; every man is a piece of the continent, a part of the main; ... and therefore never send to know for whom the bell tolls; it tolls for thee." — John Donne

## Avoidance Behavior at Breakout in COVID19

- We analyze breakout, as it resembles post lockout (low  $\pi$ )
- 1. At "breakout" of COVID19, almost everyone is susceptible.
  - Is prevalence so low that we are in the nonchalance phase?
  - ► No: Breakout growth rates are increasing in passage rates
  - ► Falling growth rates in every nation ⇒ falling passage rates
  - Behavioral SIR model explains national COVID case counts
- 2. Infected first are oblivious and noncontagious, then oblivious and contagious, then some get sick, and some don't.
  - We modify the SIR model to allow for
    - "silent spreaders" = asymptomatic infected
    - Three infected substates
  - ▶ 15% was infected in one town (Gangelt) in Germany
  - San Miguel County, Colorado has 1-3% infected
- 3. Who exactly gets tested? How does tracing help? Policy crucially depends on players' information.

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## The SI3R Model



## The SI3R Dynamic System

- In State 3, some people (fraction α₀ < 1) are asymptomatic and so unaware, or irresponsible and cavalier.
- Silent spreaders use passing rate β, and others self-quarantine, passing rate in State 3 is as if αβ (representative agent model)

$$\dot{N}_t = \sigma_t [\beta C_t^A + \alpha \beta C_t^S] - \kappa N_t \dot{C}_t^A = \kappa N_t - \psi C_t^A \dot{C}_t^S = \psi C_t^A - \nu C_t^S$$

• Example:  $R0 = \beta/\psi + \alpha\beta/\nu$ 

#### Balanced Breakout Growth Rate

- ► daily growth rate g of symptomatic incidence ψC<sup>A</sup><sub>t</sub> from asymptomatic contagious individuals.
- ▶ initially balanced contagion growth path of  $(N_t, C_t^A, C_t^S)$ , i.e., where  $\dot{N}_t = \dot{C}_t^A = \dot{C}_t^S$ .

#### Theorem (Breakout)

The initially balanced contagion growth rate g solves

$$g^{3} + (\kappa + \psi + \nu)g^{2} + [\kappa(\psi + \nu - \beta) + \nu\psi]g + \kappa(\psi\nu - \beta\nu - \alpha\beta\psi) = 0$$

▶ We can easily solve for the passing rate from the growth rate

$$\beta(g) = \frac{(g+\psi)(g+\nu)(g+\kappa)}{\kappa(g+\nu+\psi\alpha)}$$

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#### Growth Rate Against Spreading Rate



Figure: Growth Rate Against Passing Rate. From lowest to highest curves, we have silent spreader shares  $\alpha = 0, 0.1, 0.3, 0.5, 0.75, 1$ .

#1: Daily growth rates rises in the silent spreader share  $\alpha$ .#2: There are increasing returns to social distancing  $\forall g \geq \bar{q}$ .

#### "Bend the Curve" of Log-cumulative Cases $\Sigma(t)$



Figure: Curve Bending  $\Rightarrow$  U.K. Avoidance Behavior (from 100 cases)

- If  $\Sigma(t)$  has a constant growth rate, it is the slope of  $\log \Sigma(t)$ .
- Falling slope  $\Rightarrow$  falling implied passing rate!
- Best fit quadratic  $\log(\Sigma(t)) = 3.256 + 0.293t 0.002t^2$

## Testing and Tracing

Self-selection testing identifies individuals in State 3

Random testing identifies individuals in States 1,2, and State 3

- Tracing identifies individuals States 1,2, as well as asymptomatic individuals.
- Our balanced growth theorem allows us to compare the value of these policies (not yet done).

# $log(Incidence_t)$ versus $log(Prevalence_t)$ at Breakout

• A linear relationship pre-lockdown, slope  $\approx 0.9$ 

- A structural break at effective lockdown, flatter slope afterwards
- Results based on the data available at 4/9/2020 1.30pm ETS from https://github.com/CSSEGISandData/COVID-19

















# $log(Incidence_t)$ versus $log(Contagious_t)$ at Breakout

- We approximate *currently contagious* cases by the total incidence 2-17 days ago
- Test for breaks in log-log relationship 2 days after lockdown to allow the lockdown to affect spreading
- A linear relationship pre-lockdown, slope  $\approx 0.9$
- A structural break at effective lockdown, flatter slope afterwards
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Evidence of Increasing Avoidance Behavior Pre-Lockdown

Increasing avoidance behavior even pre-lockdown!

 Year-over-year decline of seated diners at restaurants prior to locally-mandated closures (from OpenTable)



## Seated Diners at Restaurants on the OpenTable Network



## Final Thoughts.

- 1. Anthony Fauci: "Models are as good as the assumptions you put into them, and as we get more data, then you put it in and that might change.
- 2. Stop using the SIR model for human pandemics.
- Official case counts are majorly undercounted! Using current death numbers, and a 1% death death, and mean about 18 days to die, and known growth rates of cases in last 18 days, imply about ten times as many cases now as claimed!
- 4. **Testing saves lives.** Each infected person is at the root of a tree of a possibly large infection tree. Until a vaccine is found, we need a national testing for entry into larger social settings.
- 5. Older tests should only work for smaller social settings. By geometry, one interacts roughly with  $O(\sqrt{n})$  people in a crowd of size n. COVID19 test should be thought as valid for two days (State 1), and then expiring linearly day by day over perhaps the next three to five days, valid for smaller n settings.