Timing Games via Nash Equilibrium

Caller Number Five:
Timing Games that Morph from One Form to Another

(Andreas Park and Lones Smith)

- Three or more players can stop at any point in time.
- Their action choice is the irrevocable stopping time $t \geq 0$. This is a nonnegative real number.
- Others' actions are unobservable, and thus there is only one information set.
- Mixed strategy: measures the probability $G(t)$ that a player has stopped by time t.
- Strategies depend simply on calendar time.

Symmetric Mixed Strategy Nash Equilibria

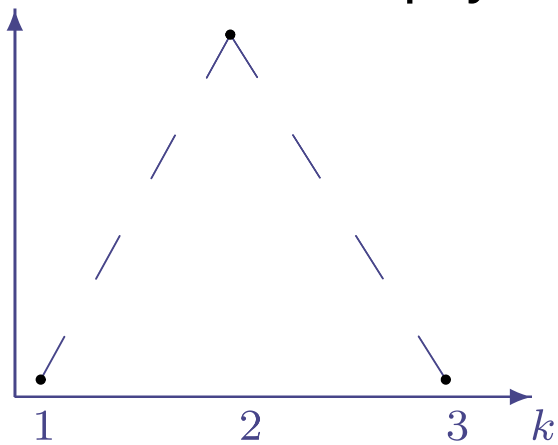
- We look for cdf's G over time $[0, \infty)$ such that all players are indifferent between entry at all moments in time.
- This has two implications:

1. Atomic entry (i.e. when the chance that any two individuals enter at a moment is positive, since the cdf G jumps): Entry at the moment before or after atom during slow play yields the same payoff.
2. slow play (i.e. when the chance that any two individuals enter at a moment is zero, because G has a density $g(t) = G'(t)$): Entry at consecutive moments in time has the same payoff.

- We now apply the solution methodology in an example.

1. How Big is the Atom?

Caller Number Two: rank payoffs $(0,1,0)$



- We guess there is smooth entry until an "atomic" entry.
- With a common entry chance G, the chances that 0 or 1 or 2 others have entered is $(1-G)^2$, $2G(1-G)$, and G^2

$\Rightarrow$ My expected payoff *before* an atom is $\phi(G) = 2G(1-G)$

- My expected payoff *in* an atom is $(1-G)^2 \cdot \frac{1}{3} + 2G(1-G) \cdot \frac{1}{2}$
- Equating $2G(1-G) = (1-G)2/3 + G(1-G) \Rightarrow G = 1/4$

2. Solving for the War of Attrition Phase

- Assume a delay cost $c(t) = t$ if one waits till time t.
- Equate marginal benefits and costs of waiting dt:

$$dt = \text{MC}(wait) = \text{MB}(wait) = \phi'(G(t))dG$$

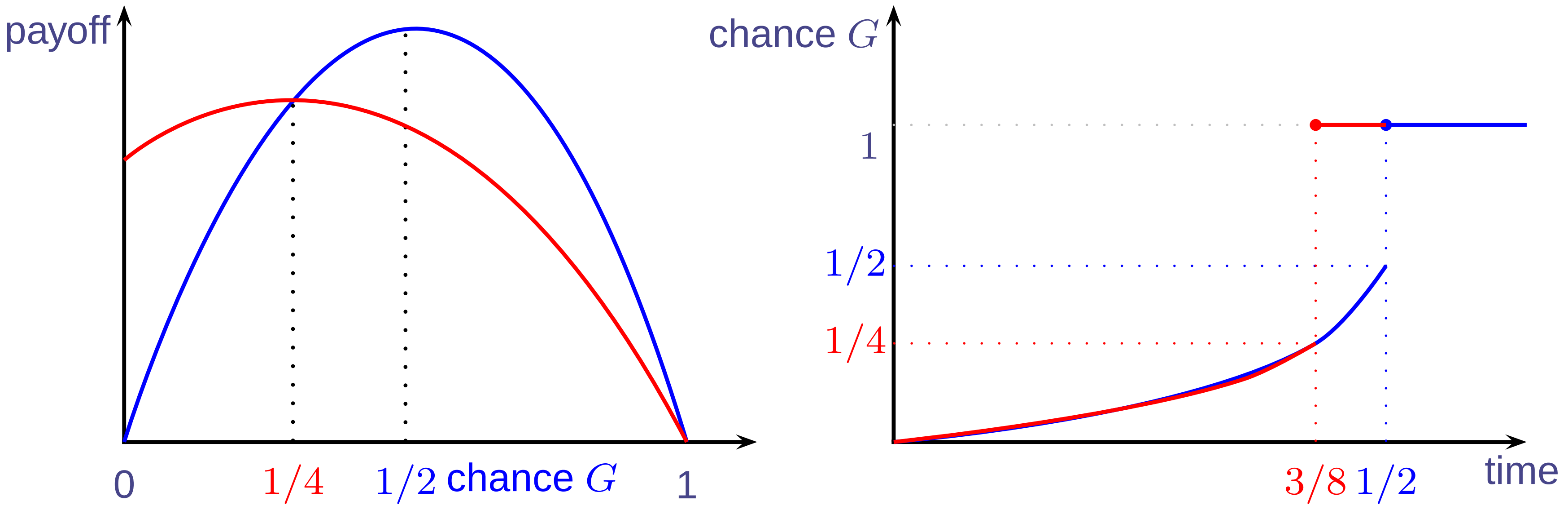
$$\Rightarrow 1 = \phi'(G(t))\dot{G} = (2 - 4G(t))\dot{G}(t)$$

- If we do not start with an atom, then $G(0) = 0$.

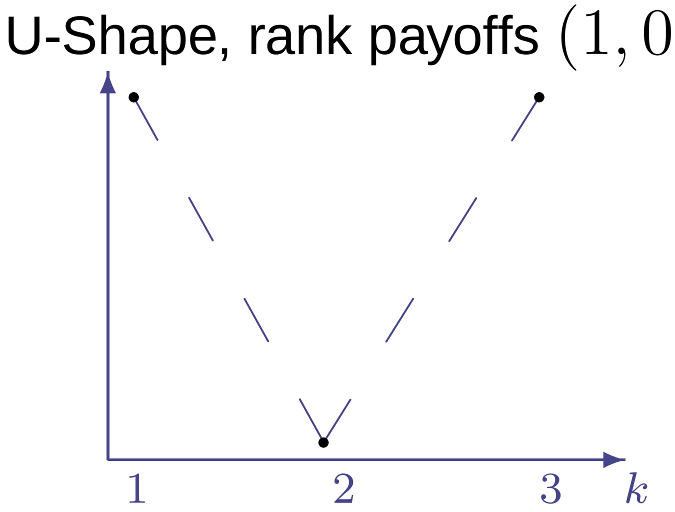
$$\Rightarrow G(t) = 1/2 - 1/2\sqrt{1 - 2t}.$$

- This is well-defined until $t = 1/2$, when $G = 1/2$.
- Know atom occurs when $G = 1/4$
  $\Rightarrow$ smooth play until $G(t) = 1/4$ at time $t = 3/8$.
- the game ends with a complete atom.

Graphs for Caller Number 2 of 3



Example 2: U-Shape



- Players want to be first or last.
- Urge to go for being first $\Rightarrow$ expect time-0 atom.
- Atom size $G \Rightarrow$ get $1, 1/2, 2/3$ with chance $(1-G)^2, 2G(1-G)$ and G^2 respectively.

Example 2: How big is the atom?

- $\Rightarrow$ expected payoff *in* an atom is $(1-G)^2 + G(1-G) + 2G^2/3$.
- Expected payoff *after* atom is $\phi(g) = (1-G)^2 \cdot 1 + G^2 \cdot 1$.
- Equating $(1-G)^2 + G(1-G) + 2G^2/3 = (1-G)^2 + G^2$ $\Rightarrow G = 3/4$.

2. Solving for the War of Attrition Phase

- As above assume a delay cost $c(t) = t$ if one waits till time t.
- Equate marginal benefits and costs of waiting dt:

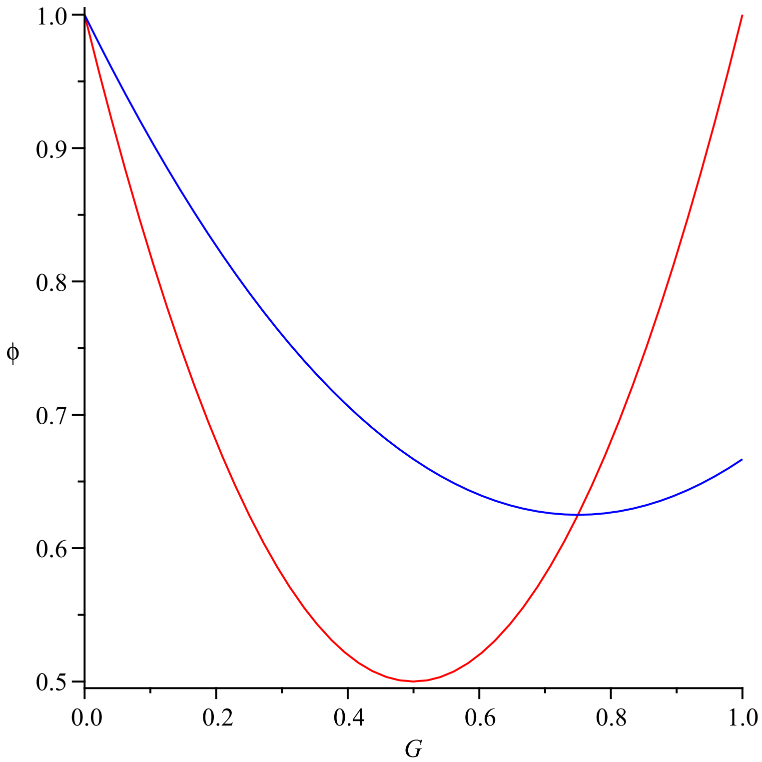
$$0 \;=\; -1 - \dot{G}(t)(2 - 4G(t))$$

- Since we start If we do not start with an atom, then $G(0) = 3/4$.

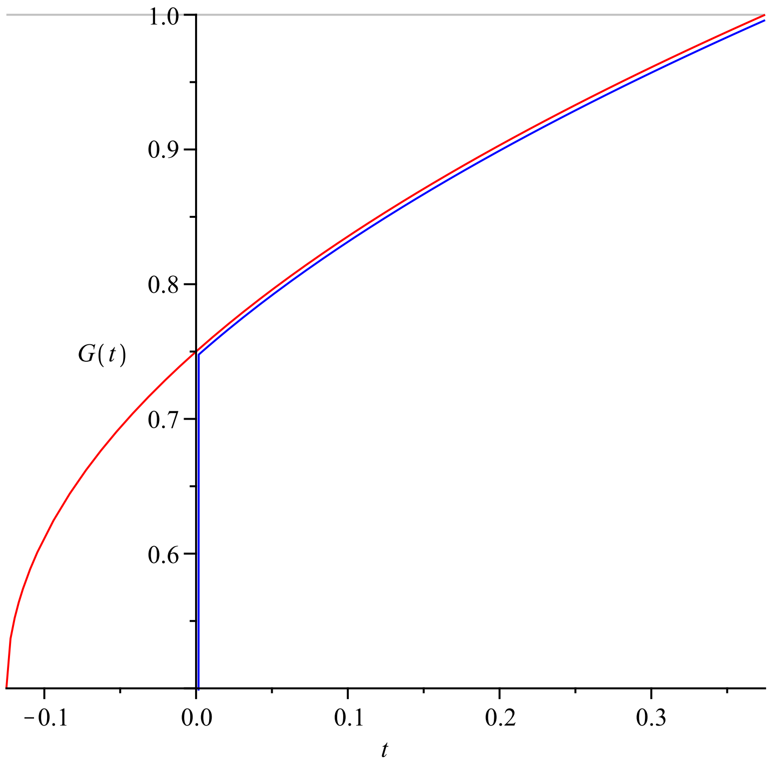
$$\Rightarrow G(t) = 1/2 + 1/2\sqrt{1/4 + 2t}.$$

- This is well-defined for all $G > 1/2$.
- Know atom $G = 3/4$ occurs at $t = 0 \Rightarrow$ smooth play from $t = 0$ until time $t = 3/8$ when $G(3/8) = 1$.

Equilibrium Play with U-Shape



Expected payoffs ϕ and payoff from atom



Equilibrium strategy (blue) and possible range for G